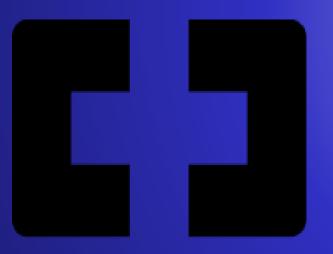
# A Unified Framework for Sequential Decisions under Uncertainty



#### Warren B Powell

Chief Analytics Officer, Optimal Dynamics Professor Emeritus, Princeton University

> Institute of Data Sciences Texas A&M University January 24, 2022



### Laboratory sciences

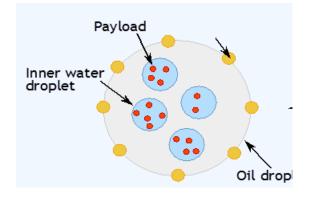
Materials science

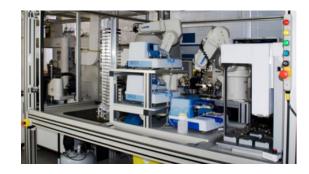
**Optimal Dynamics** 

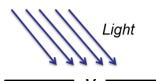
 » Optimizing payloads: reactive species, biomolecules, fluorescent markers, ...

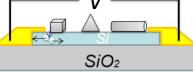
 » Controllers for robotic scientist for materials science experiments

 » Optimizing nanoparticles to maximize photoconductivity





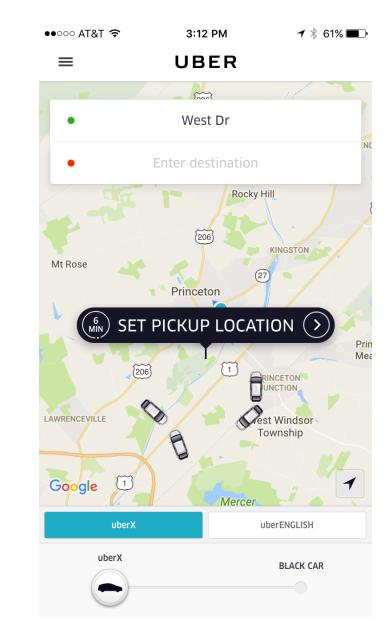






# Managing ride-hailing fleets

- Uber
  - Provides real-time, on-demand transportation.
  - Drivers are encouraged to enter or leave the system using pricing signals and informational guidance.
- Decisions:
  - » How to price to get the right balance of drivers relative to customers.
  - » Assigning and routing drivers to manage Uber-created congestion.
  - » Real-time management of drivers.
  - » Pricing (trips, new services, ...)
  - Policies (rules for managing drivers, customers, ...)





# Managing ride-hailing fleets

 Now we have a logistic curve for each origin-destination pair (i,j)

$$P^{Y}(p,a \mid \theta) = \frac{e^{\theta_{ij}^{0} + \theta_{ij}p + \theta_{ij}^{a}a}}{1 + e^{\theta_{ij}^{0} + \theta_{ij}p + \theta_{ij}^{a}a}}$$

- Number of offers for each (i,j) pair is relatively small.
- Need to generalize the learning across hundreds to thousands of markets.



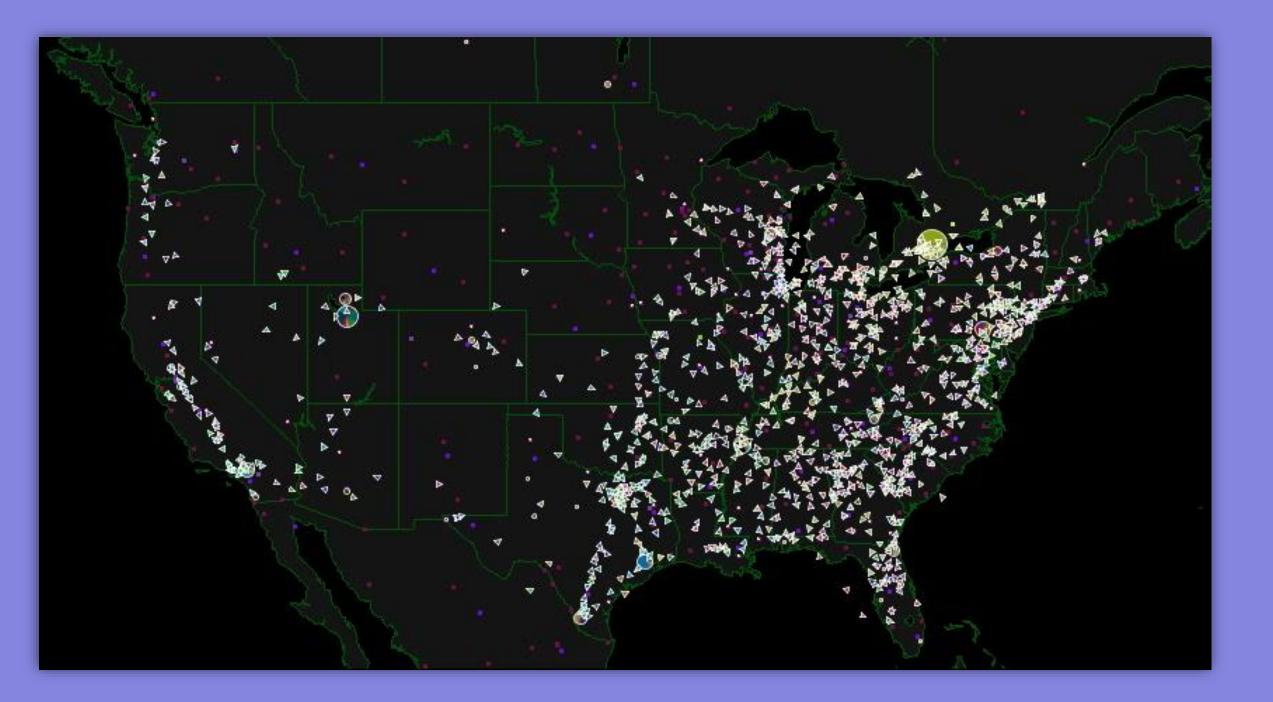


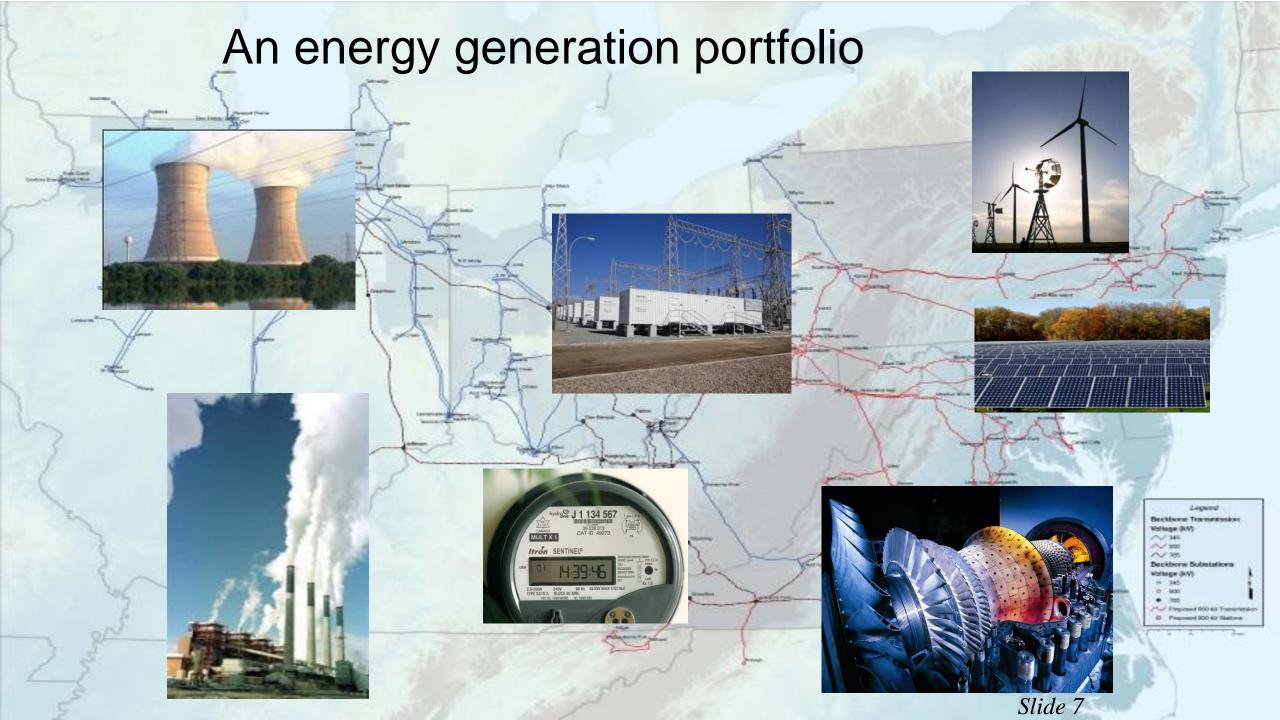
### **Fleet management**

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- Fleet management problem
  - » Optimize the assignment of drivers to loads over time.
  - » Tremendous uncertainty in loads being called in

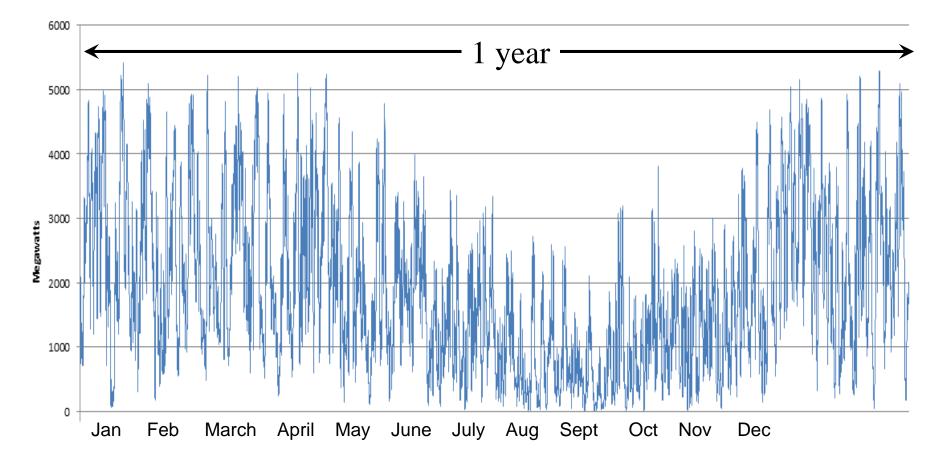






### Energy from wind

### □ Wind power from all PJM wind farms



#### Sensing

- » Drones fly over the ocean to detect the presence of oil.
- Communication
  - » Drones communicate by sharing information

#### Mitigation

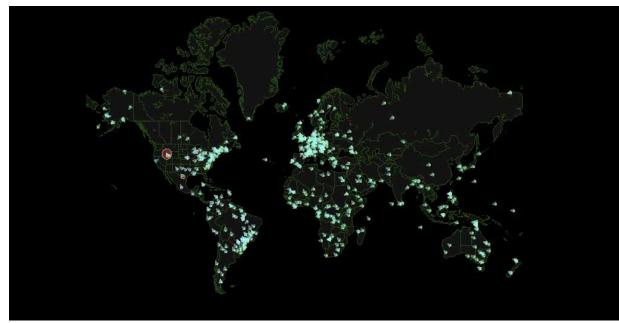
 Information from drones can be used to guide surface vehicles performing cleanup.

# Multiagent supply chain management



- Managing the supply chain
  - Challenge is determining when to order parts given the long lead times, and production uncertainties.
  - » Suppliers work for multiple customers.

- Pratt & Whitney jet engines
  - » Over 1,000 parts
  - » Median lead time for a part is 120 days. Some lead times are over 300 days.
  - » Parts often require reworking.

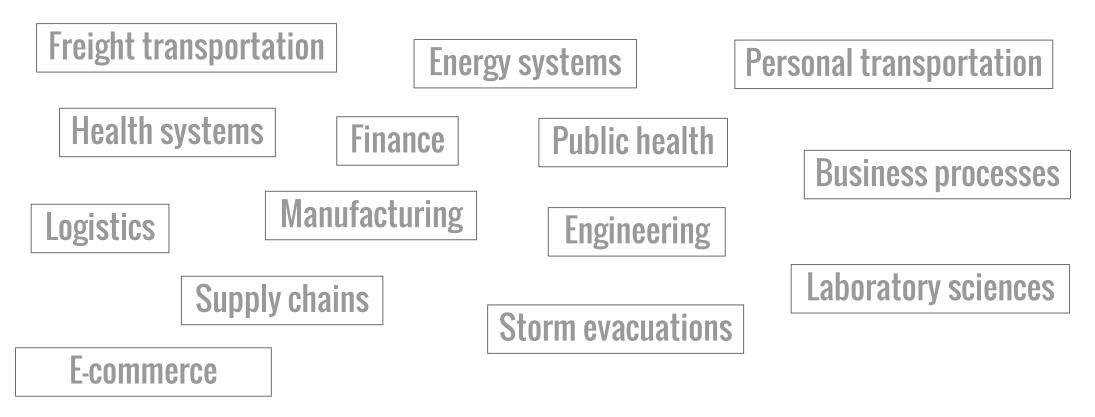






### CHALLENGES

We are looking for opportunities for making better decisions where we have to deal with uncertainty





### **GOALS & OBJECTIVES**

- » Reduce costs
- » Increase profits
- » Improve reliability
- » Minimize waste

**Optimal Dynamics** 

» Maximize strength

- » Improve health
- » Reduce risk
- » Increase yield
- » Reduce carbon production
- » Minimize lives lost



### **GOALS & OBJECTIVES**

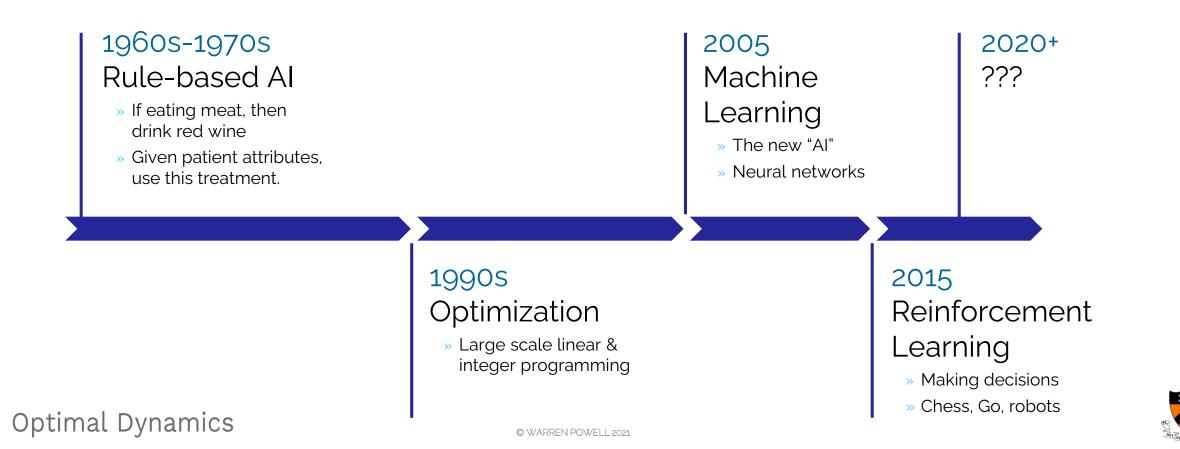
Improve performance by making better decisions.





### **Intelligent decisions**

Artificial Intelligence
1. Making computers behave like humans
2. Making computers smarter than humans



# WHAT'S NEXT IN AI?

*Sequential decision problems*, where we need to make decisions over time, as new information arrives.

I propose to unify 15 distinct fields that deal with dynamic decision making into a new field that I call *Sequential Decision Analytics*.

Sequential decision analytics includes all of reinforcement learning, but is broader, with a greater emphasis on uncertainty.



### OUTLINE

→ The five layers of intelligence

- → Modeling sequential decision problems
- → Designing policies
  - → Policy function approximations
  - → Cost function approximations
  - $\rightarrow$  Value function approximations
  - → Direct lookahead approximations

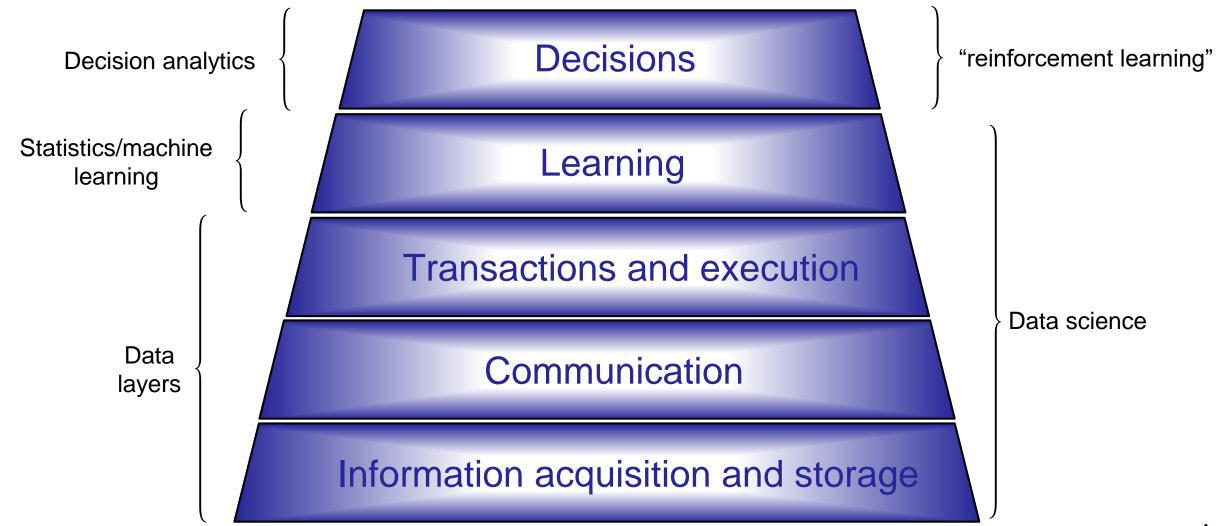
→ A new educational field: sequential decision analytics

### OUTLINE

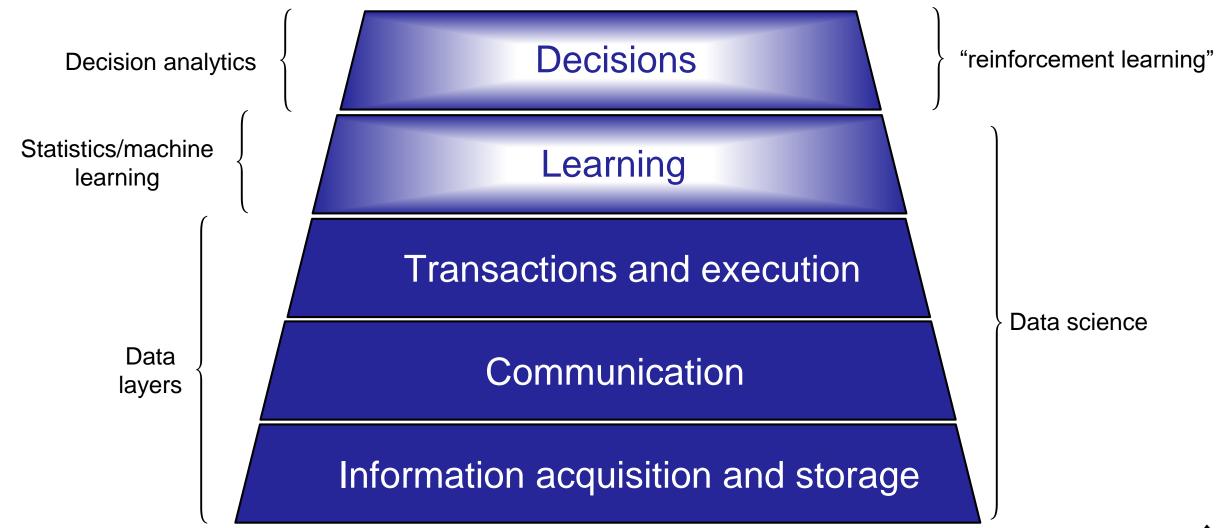
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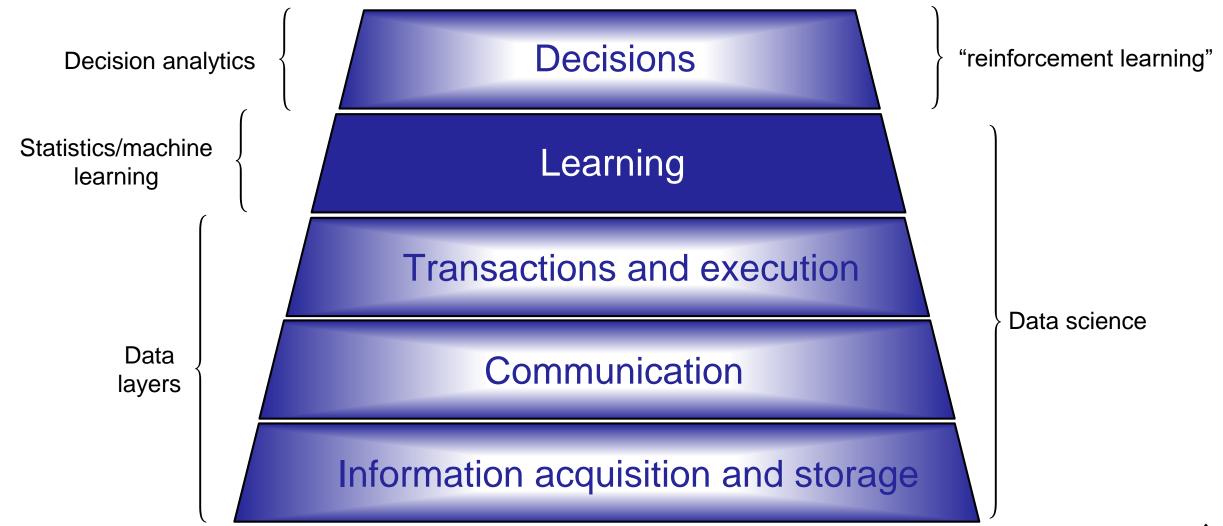
→ A new educational field: sequential decision analytics













# **MACHINE LEARNING**

### Types of Learning

### Pattern Matching

### Classification

#### Inference

#### Prediction



- » What is the voice saying?
- » What is in the picture?
- » What is the email asking for?

- » What product should I recommend for this customer?
- » What treatment should I recommend for this patient?

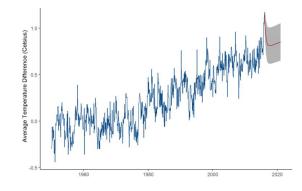
» How will an increase in

» What is the condition of a

piece of equipment?

price affect market

demand?



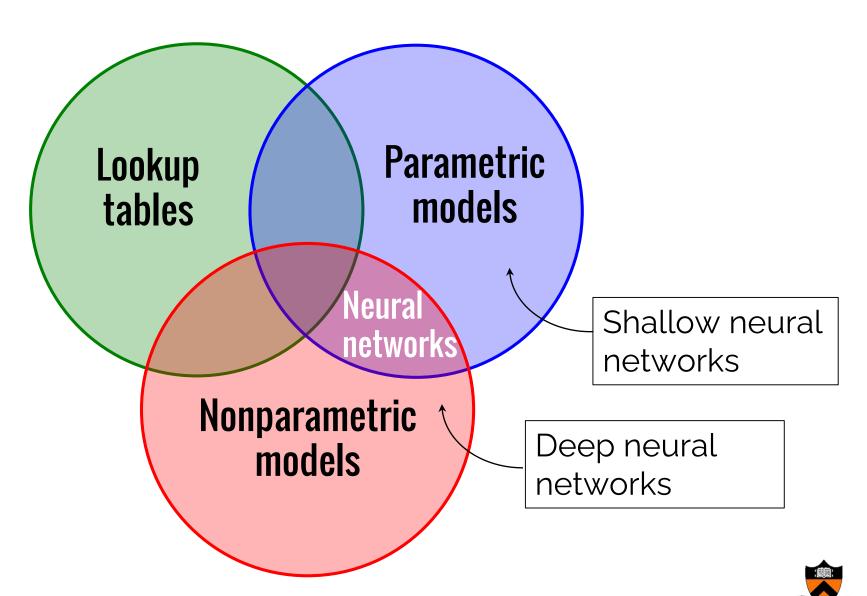
- » What will the market demand be in three days?
- » How many loads will the shipper need to move in a week?





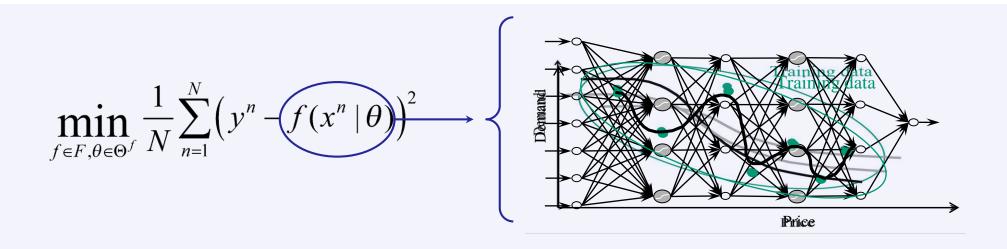
### **MACHINE LEARNING**

Every single machine learning method falls in one of these three circles.



### BRIDGING MACHINE LEARNING & SEQUENTIAL DECISIONS

Machine learning as an optimization problem

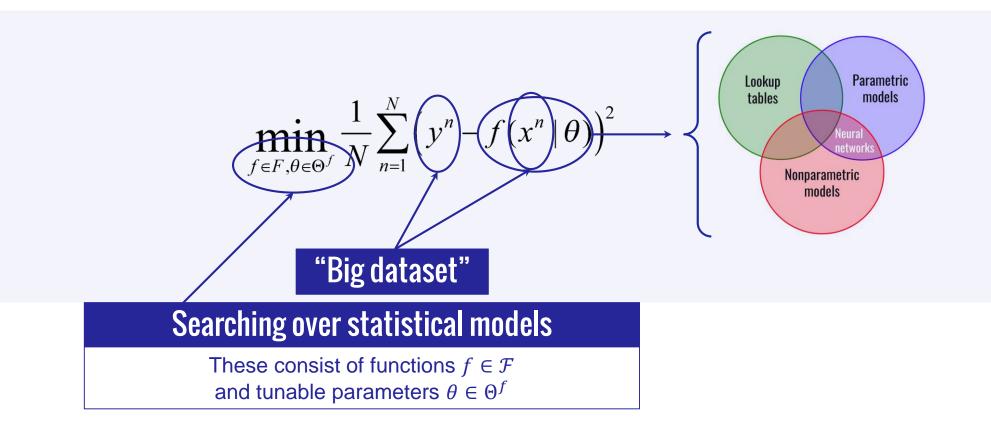


The first step is choosing a mathematical model that will do the best job of fitting the data (but be careful of overfitting with neural networks).



### BRIDGING MACHINE LEARNING & SEQUENTIAL DECISIONS

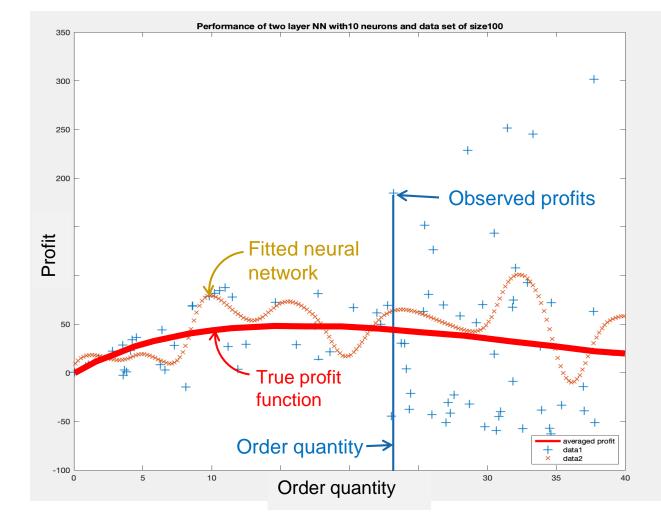
Machine learning as an optimization problem





# **MACHINE LEARNING**

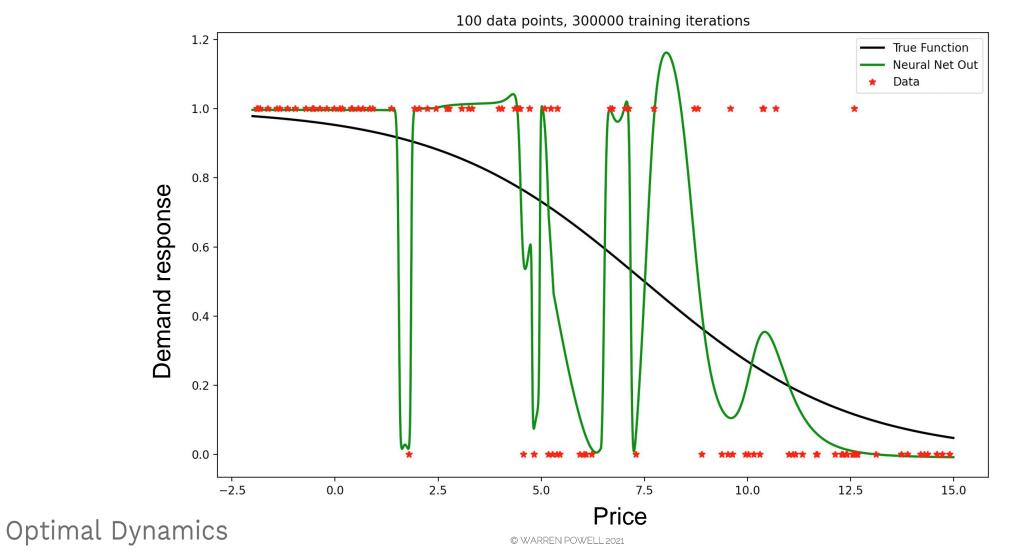
- Neural networks struggle with:
  - Noise Their high flexibility tends to fit the noise.
  - Structure It is difficult to communicate structure:
    - Monotonicity Higher price means lower demand
    - Concavity As with the newsvendor problem to the right.



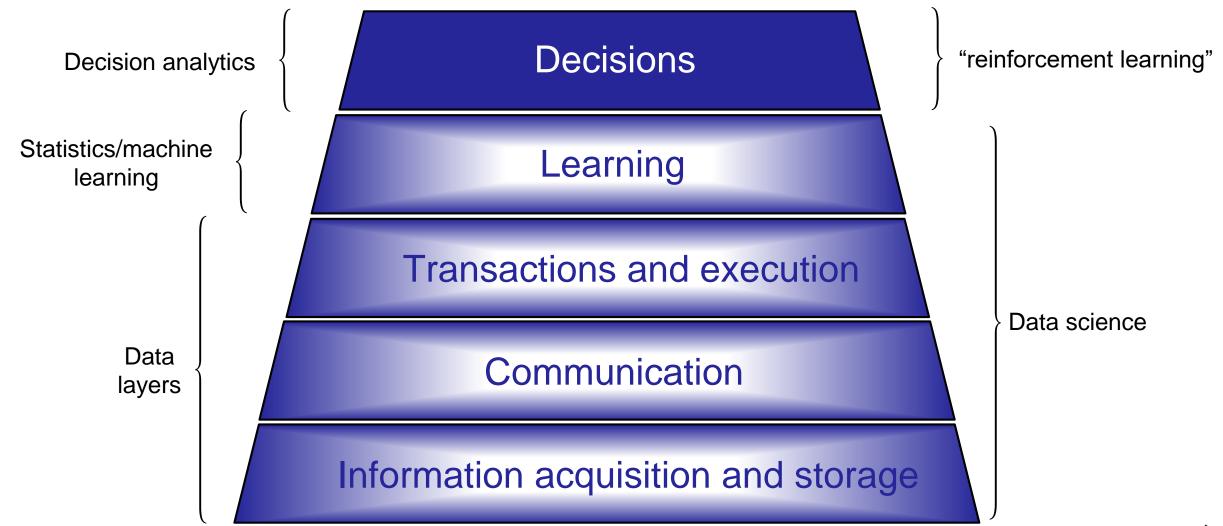


### **MACHINE LEARNING**

> Neural network for demand response:









# **INFORMATION & DECISION PROCESSES**





### Information and decision processes

 There are parallels between the process of making "decisions" and a manufacturing line making "products"



() Optimal Dynamics approach information processing and decisions like a manufacturing process.





# DECISIONS

		What contracts to sign for
What price to accept for a spot load?	Which driver should What is the best policy for	raw materials?
Which load to accept now	move a load? high-frequency trading?	Which vendor should
to move next week	edicated How many syringes should financial op	
Where should drivers should	d we have	When should inventory
drivers be	How much batt	
domiciled? Which phys	sician should rocedure? How many nurses variability of wir	what price should
When should I refill the custon	should we have to	be charged
with liquid nitrogen	Which nurse should visit this scheduled	d to handle drops in
Which customer tanks should		
we fill when we are in the are	- Where should a patient be II How many su	ppliers should you have for
Which material handling jobs	assigned for specific treatment? a particular particula	art, and where?
should be done by robots, and	What bid should we place on Google for a set of ad-words? How many aircraft	Which supplier should
which robot?	Google for a set of ad-words?	manufacture turbine blades?
When should inventory be $\vee$	Which fulfillment center	How many jet engines should
······································	hould handle an order?	be made each day?
() Optimal Dynamics	© WARREN POWELL 2021	BELEVIS MANNETURET

# DECISIONS

### Types of decisions.

#### **Physical Decisions**

#### Financial Decisions

#### Informational Decisions



K STOCK EXCHANGE

- » Managing inventories
- » Assigning drivers and moving trucks
- » Scheduling nurses and energy generators

- » Pricing decisions
- » Insurance decisions
- » Managing investments
- » Hedging contracts



- » Sending/receiving information
- » Marketing and advertising
- » Running experiments (lab or field)
- » Testing drugs



# THE TIME FRAMES FOR DECISIONS

**Strategic planning and design** – We simulate operational decisions so we understand how a system would respond to decisions far in the future:

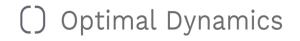
- » How many gas turbines should a power grid have?
- » How should we design a building to withstand earthquakes?
- » What should the capacity of a levee or reservoir be?

**Tactical planning decisions** – We simulate operational decisions to help make decisions that impact the system in the near future,

- » How much energy generation should the grid plan for tomorrow?
- » How many gallons of water should be ordered in anticipation of a hurricane?
- » How to allocate traffic management personnel to handle storm evacuations?

#### **Real-time decisions** – These are decisions that impact the system now:

- » Making real-time ramping decisions for energy generators.
- » Notifying houses within a zone to begin evacuations before a storm.
- » Operating pumps to mitigate flooding during a storm.





### THE TIME FRAMES FOR DECISIONS

**Strategic planning and design** – We simulate operational decisions so we understand how a system would respond to decisions far in the future:

We need to simulate decisions in the future that **do** *not* depend on the state of the system now.

**Tactical planning decisions** – We simulate operational decisions to help make decisions that impact the system in the near future,

We need to simulate decisions in the future that *do* depend on the state of the system now.

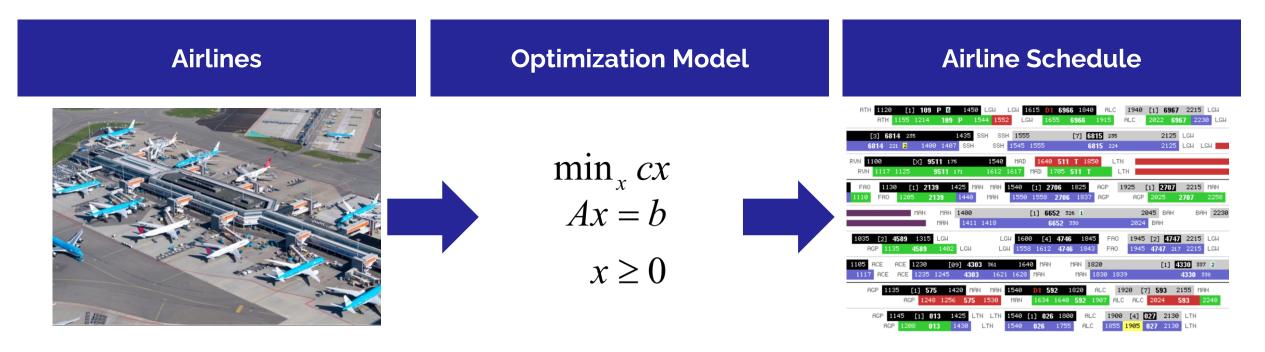
**Real-time decisions** – These are decisions that impact the system now:

We need to simulate the effect of a decision now (which depends on the state of the system) on the future.



# **DETERMINISTIC OPTIMIZATION**

### Airline scheduling



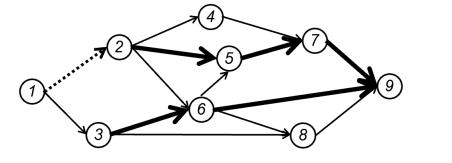
Airlines around the world use tools that depend on this mathematical model to perform strategic and operational planning.



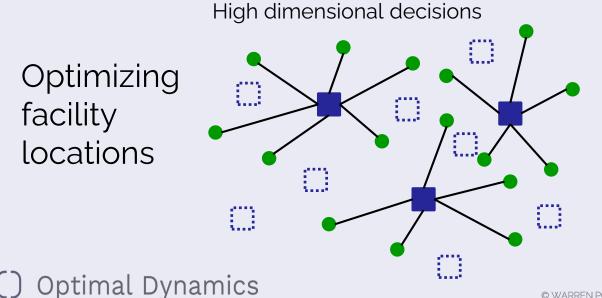
## **DETERMINISTIC OPTIMIZATION**

Low dimensional decisions

Planning a path to your destination



 $x_{ij} = \begin{cases} 1 & If we move from node i to node j \\ 0 & Otherwise \end{cases}$ 



 $x_i = \begin{cases} 1 & If we locate a facility at location i \\ 0 & Otherwise \end{cases}$ 



In most settings, decisions are made over time...

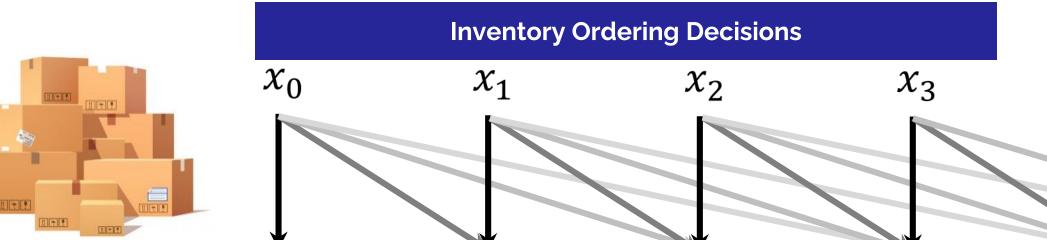


Information that arrives after a decision is not known when we made the decision.



Optimal Dynamics

#### Inventory management



#### **Customer Demands (information)**

#### () Optimal Dynamics

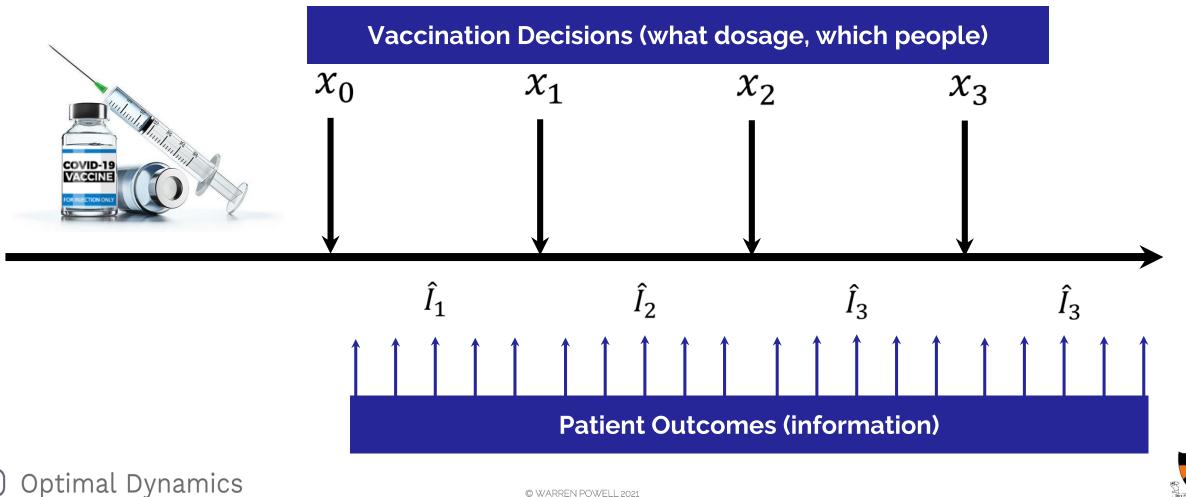


#### Driver dispatch for truckload trucking

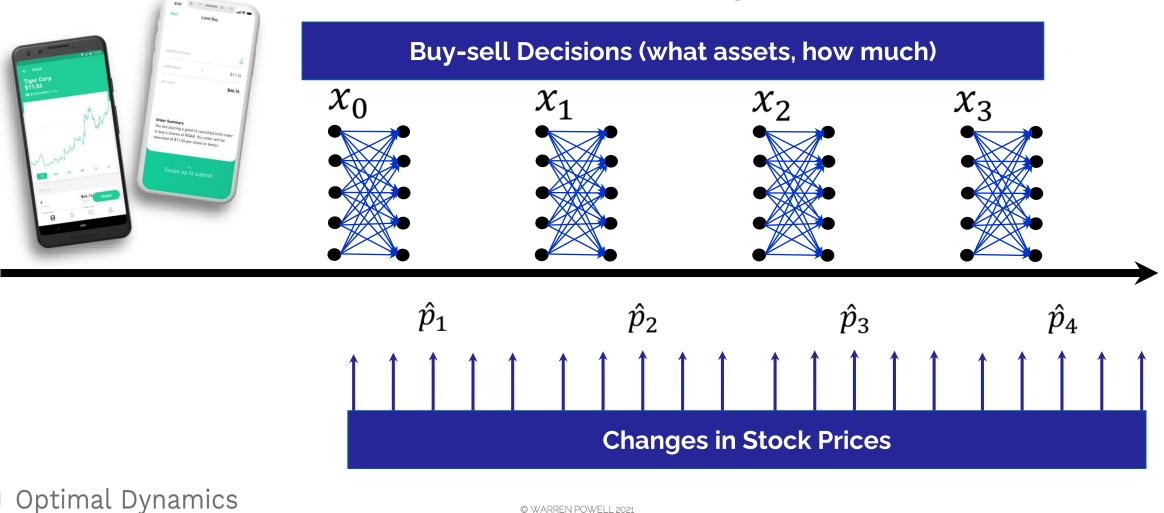
**Decisions Assigning Drivers to Loads**  $x_0$  $x_3$  $x_2$ ·····> .....> •••••  $\widehat{D}_1$  $\widehat{D}_2$  $\widehat{D}_3$  $\widehat{D}_4$ Shippers Calling in Loads (information)

() Optimal Dynamics

Testing new vaccines



#### **Financial Trading**



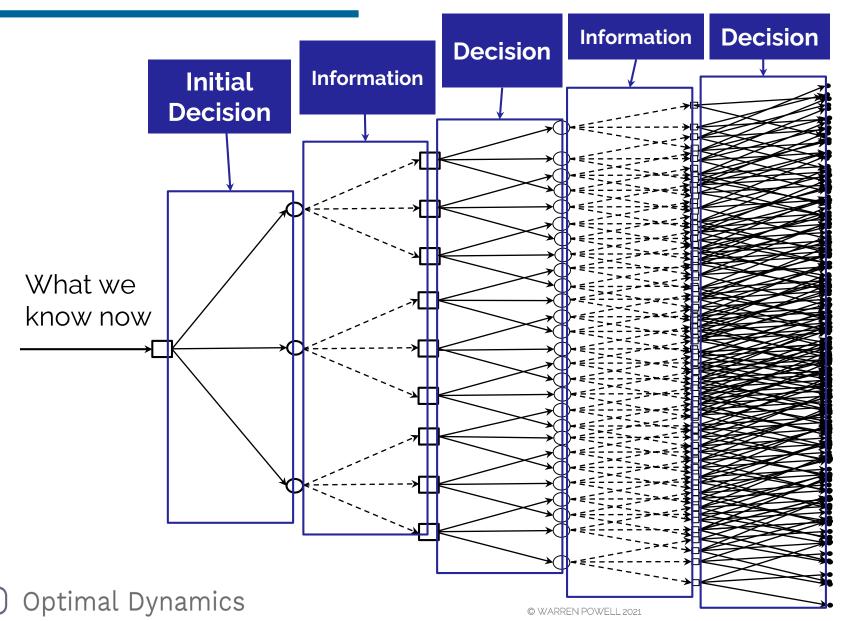


# DECISIONS

What price to accept for a		What contracts to sign for raw materials?
spot load? Which load to accept now to move next week	high-free	equency trading? Which vendor should supply each part?
Where should drivers should drivers be domiciled:		should financial option when should inventory
•	ysician should How many nurs procedure? should we have	rses needed to handle the variabili be charged
When should I refill the custor with liquid nitrogen	doctor's offices	s? When should gas turbines be scheduled to handle drops in
Which customer tanks shoul fill when we are in the area?	d we Which nurse sho doctor's office to Where should a patient be	odav?
Which material handling jobs	assigned for specific treatme	ent? How many suppliers should you have for a particular part, and where?
should be done by robots, and which robot?	What bid should we place on Google for a set of ad-words?	How many aircraft should I order for
When should inventory be Which fulfillment center refilled at a fulfillment center? handle an order?		delivery in five years? How many jet engines should be made each day?
J Optimal Dynamics	© WARREN POWELL 2021	Burling municipal

# INFORMATION

Market prices for	spot freight		quests for	1		Pi	rices of raw	materials by regior	า า
Offered loads by shi	pper. by lan	<u>lo</u> ads; tim	ie-at-home	Changes in a	· · · · · · · · · · · · · · · · · · ·		nrc	ality of orders ovided by a vendor	
Driver application	Employmer unemploym	nt rate;	New COVID county	-19 cases by	order f	ulfillm	lelays in pro	-	_
for jobs by region		rivals and	Reques	sts for nurses	U	eratior	n from a win	Competitor prices	5
Customer usage	rate of liqui	d nitrogen	from do	octor's office	ès [	Elect	ricity prices	on the grid	
Equipment failu	ures at custo	omer	Number of n	urses calling	g in sick				
nitrogen tanks Flow of different parts to each machining station		Availability of specialists to tr			eat Capacity shutdowns at suppliers due to labor or political problems				
		Whether a bid wins an ad-click auction			Availability of		Lead times hanufactur	required by each er	
Flow of orders for a region around the co	ountry	Orders for different re	a product fro egions	· · ·	oduction cap new jets	1	Daily produ engines	ction of new jet	
() Optimal Dynar	11105		© WARREN	POWELL 2021				DELEVE WAINT	IVIGET



Even small sequential decision problems explode dramatically as we plan into the future



#### OUTLINE

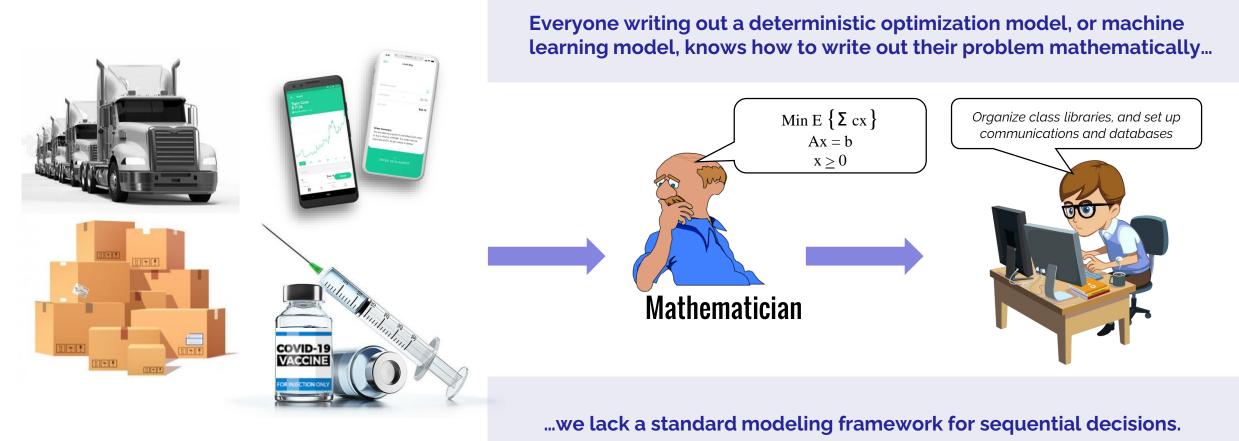
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  - → Direct lookahead approximations

→ A new educational field: sequential decision analytics

## **MODELING SEQUENTIAL DECISION PROBLEMS**

The biggest challenge when making decisions under uncertainty is modeling.



#### () Optimal Dynamics



Stochastic programming

Simulation optimization

Optimal Bandit learning problems Model predictive

Active learning

> Reinforcement learning

control

Stochastic control

Robust optimization

Optimal

control

Markov

decision

processes

Dynamic Programming and control

ecision

analysis

Online computation

Simulation optimization

Stochastic

search

Approximate

dynamic

programming

© Warren Powell 2021



• Any sequential decision problems can be written:

$$(S_0, x_0, W_1, S_1, x_1, W_2, \dots, S_t, x_t, W_{t+1}, S_{t+1}, \dots, S_T)$$
  
What we know (or believe)  
What we observe (or learn)  
The decision

- Each time we make a decision, we receive a contribution  $C(S_t, x_t)$ .
- Decisions are made with a method or policy  $X^{\pi}(S_t)$ .
- The goal is to find the policy that maximizes expected contributions:

$$\max_{\pi} \mathbb{E}\left\{\sum_{t=0}^{T} C\left(S_{t}, X^{\pi}(S_{t})\right) | S_{0}\right\}$$

() Optimal Dynamics

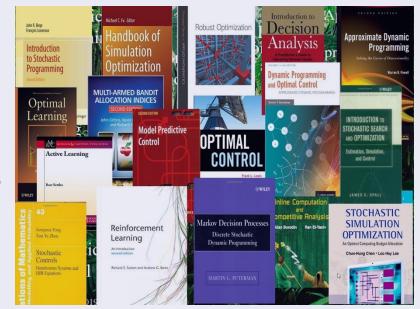


# **MODELING SEQUENTIAL DECISION PROBLEMS**

Every sequential decision problem can be modeled using 5 core components

» State variables  $S_t = (R_t, I_t, B_t)$ 

- Physical state  $R_t$ , other information  $I_t$ , belief state  $B_t$ .
- » Decision variables  $(x_t, a_t, u_t)$ 
  - Made with *policy*  $X^{\pi}(S_t|\theta)$  (or  $A^{\pi}(S_t)$  or  $U^{\pi}(S_t)$ )
- » Exogenous information  $W_{t+1}$ 
  - What do we learn for the first time between t and t + 1?
- » Transition function  $S_{t+1} = S^M(S_t, x_t, W_{t+1})$ 
  - How do the state variables evolve over time?
- » Objective function
  - $\max_{\pi} \mathbb{E}_{S_0} \mathbb{E}_{W_1, \dots, W_T \mid S_0} \sum_{t=0}^T C(S_t X^{\pi}(S_t))$



These five elements describe any sequential decision problem.



# Modeling supply chain problems

#### We start by identifying:

- » What are the performance metrics you are focusing on?
- » What decisions are involved?
- » What are the sources of uncertainty and new information?
- » What information is needed to compute metrics, make decisions, and model their evolution over time ("state variables")



#### OUTLINE

→ The five layers of intelligence

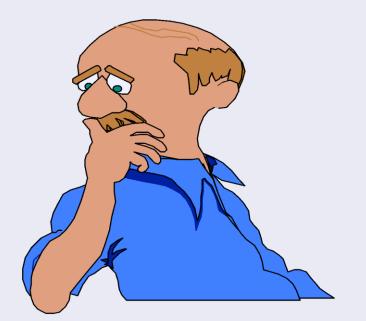
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→ A new educational field: sequential decision analytics

#### **Policy Definition**

**Optimal Dynamics** 

A policy is method that maps a state variable into a decision ... *any method.* 





#### Policies and the English language

Algorithm	Format	Prejudice
Behavior	Formula	Principle
Belief	Grammar	Procedure
Bias	Habit	Process
Canon	Laws/bylaws	Protocols
Code	Manner	Recipe
Commandment	Method	Ritual
Conduct	Mode	Rule
Control law	Mores	Style
Convention	Orthodox	Syntax
Culture	Patterns	Technique
Customs	Plans	Template
Duty	Policies	Tenet
Etiquette	Practice	Tradition
Fashion	Precedent	Way of life

() Optimal Dynamics

#### http://tinyurl.com/policiesanddecisions



# **Designing policies**

Every sequential decision problem can be modeled using 5 core components

- » State variables  $S_t = (R_t, I_t, B_t)$ 
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**Optimal Dynamics** 

•  $\max_{\pi} \mathbb{E}_{S_0} \mathbb{E}_{W_1, \dots, W_T \mid S_0} \sum_{t=0}^T C(S_t, X^{\pi}(S_t))$ 



These five elements describe any sequential decision problem.



# **Evaluating policies**

#### 1) Theoretically

- Optimality proofs
- Regret bounds
- Asymptotic convergence

#### 2) Through numerical simulations



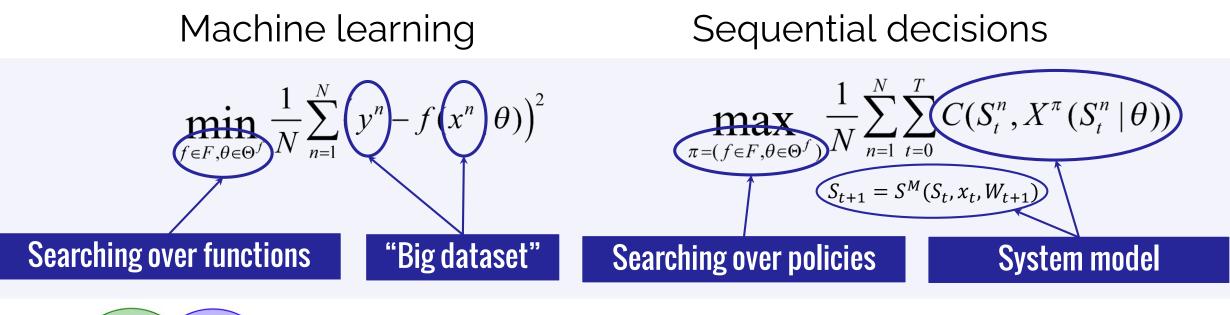
#### 3) In the field

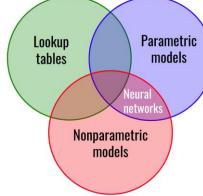






#### BRIDGING MACHINE LEARNING & SEQUENTIAL DECISIONS





Optimal Dynamics

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Two fundamental strategies for designing policies

**Policy search** – Search over a class of methods for making decisions to optimize some metric over time.

- » Finding the best class of policy.
- » Finding the best policy within the class.

**Lookahead approximations** – Approximate the impact of a decision now on the future.

- » The contribution from the first period, plus
- » An approximation of the sum of contributions in future time periods resulting from the first decision.



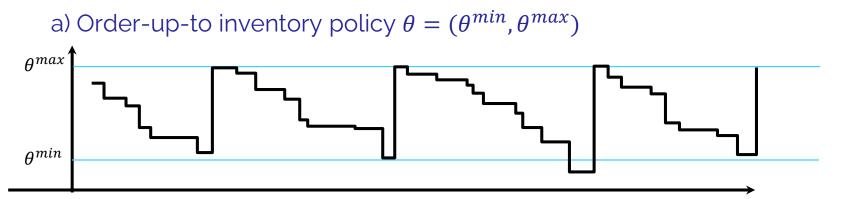
() Optimal Dynamics

Policy search

#### 1) Policy function approximation (PFA)

These are analytical functions that specify what to do given what we know.

Examples:



- b) Buy when the price goes **below**  $\theta^{min}$ and sell when it goes **above**  $\theta^{max}$
- c) Lookup tables, linear/nonlinear models, neural networks, nonparametric models, ...

DELE SYS WARDE

#### () Optimal Dynamics

Policy search

#### 2) Cost function approximations (CFAs)

These are parameterized optimization problems:

- a) Find the shortest path to a destination, but add a buffer  $\theta$  (e.g. 15 minutes) to make sure you arrive on time.
- b) Optimize energy generation for tomorrow to meet forecasted demand, but add reserves  $\theta$ in case of a generator failure.
- c) Advertise the product x which solves:

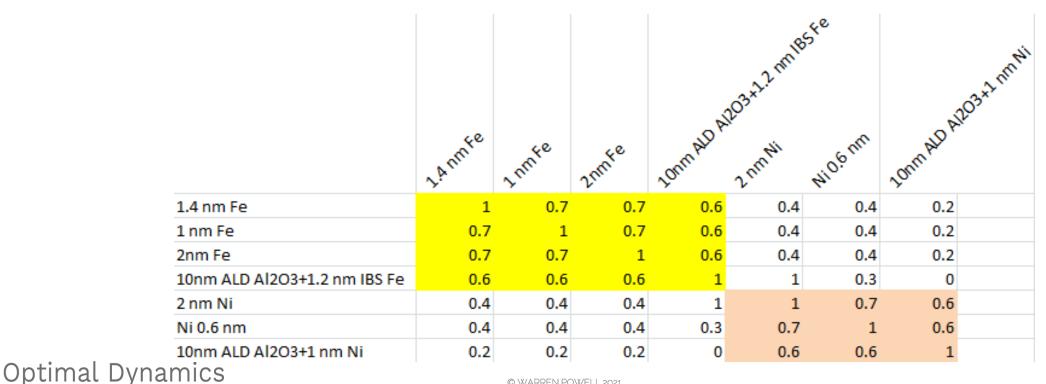
 $X^{UCB}(S^n|\theta) = \max_{x}(Estimated \ revenue_x^n + \theta \cdot Standard \ deviation \ of \ estimated \ revenue_x^n)$ 

Now solve:  $\max_{\theta} \mathbb{E}\{\sum_{n=1}^{N} C(S^{n}, X^{\pi}(S^{n}|\theta)) | S_{0}\}$ 

Parametric CFAs are widely used in industry, yet dismissed by the academic research community. This is actually quite a powerful strategy. **Optimal Dynamics** 

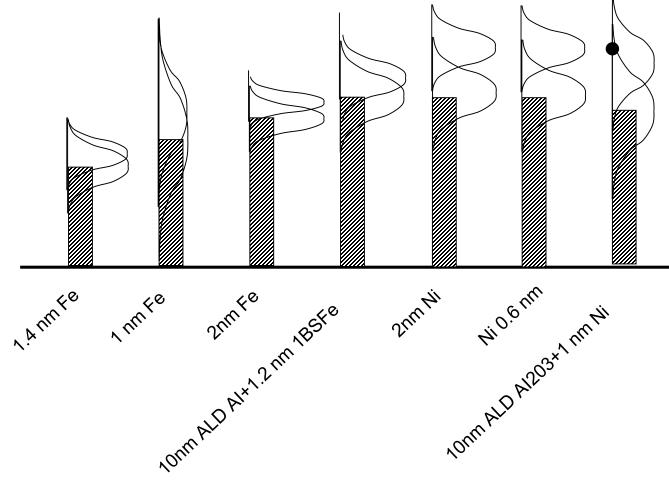


- Lookup table •
  - We can organize potential catalysts into groups  $\gg$
  - Scientists using domain knowledge can estimate correlations in >> experiments between similar catalysts.

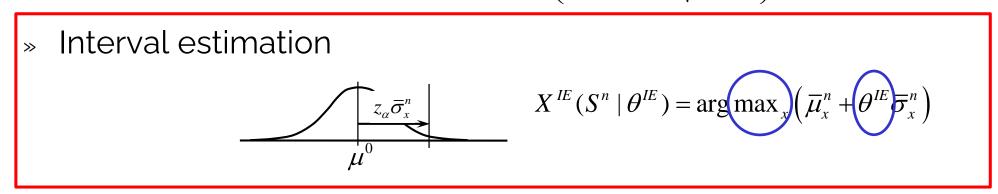




• Correlated beliefs: Testing one material teaches us about other materials



- Cost function approximations (CFA)
  - » Upper confidence bounding  $X^{UCB}(S^n | \theta^{UCB}) = \arg(\max_x) \left( \overline{\mu}_x^n + \theta^{UCB} \right) \left( \frac{\log n}{N^n} \right)$



» Thompson sampling

$$x^n = argmax_x \hat{\mu}_x^n$$

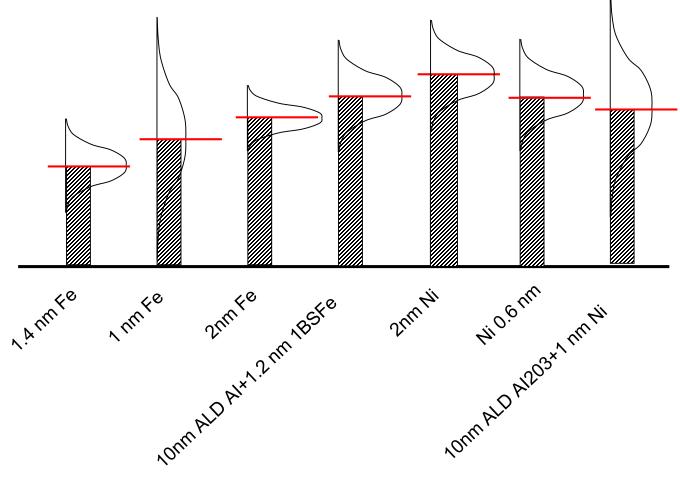
$$\hat{u}_x^n \sim N(\bar{\mu}_x^n, \theta^{TS} \bar{\sigma}_x^{2,n})$$



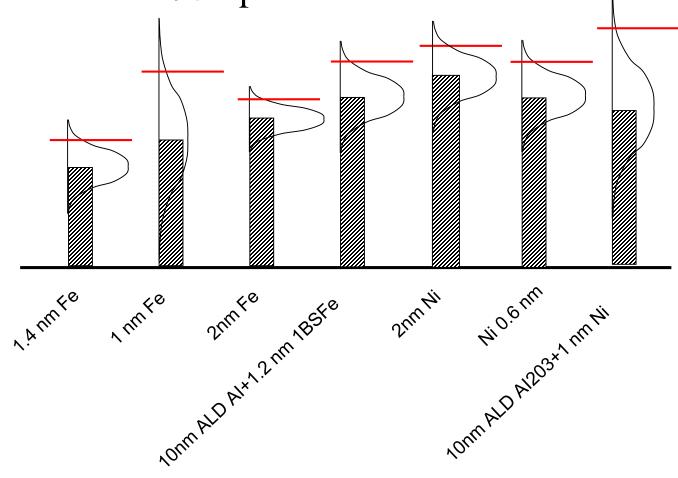
() Optimal Dynamics

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• Picking  $\theta^{IE} = 0$  means we are evaluating each choice at the mean.



• Picking  $\theta^{IE} = 2$  means we are evaluating each choice at the 95<sup>th</sup> percentile.



- Optimizing the policy ٠
  - » We optimize  $\theta^{IE}$  to maximize:

$$\max_{\theta^{IE}} F(\theta^{IE}) = \mathbb{E}F(x^{\pi,N}, W)$$

where

$$x^{n} = X^{IE}(S^{n} | \theta^{IE}) = \arg\max_{x} \left(\overline{\mu}_{x}^{n} + \theta^{IE}\overline{\sigma}_{x}^{n}\right) \qquad S^{n} = (\overline{\mu}_{x}^{n}, \overline{\sigma}_{x}^{n})$$

- Notes: ٠
  - This can handle any belief mode for the including correlated beliefs, nonlinear belief models. All we require is that we be able to simulate a policy. ≫
  - $\gg$

IE KG 2.5 3.5 0.5 1.5 2 3 'n IE parameter  $\theta^{\rm \tiny IE}$ 



#### **Optimal Dynamics**

An energy storage problem:

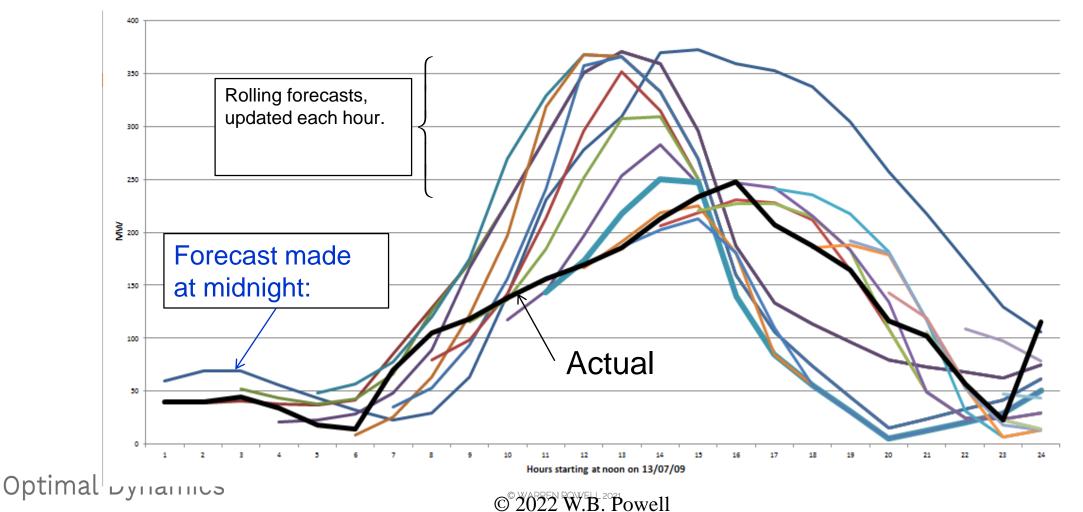




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 Forecasts evolve over time as new information arrives:





Benchmark policy – Deterministic lookahead

$$X_{t}^{D-LA}(S_{t}) = \arg \min_{x_{t}, (\tilde{x}_{tt'}, t'=t+1, \dots, t+H)} \left( C(S_{t}, x_{t}) + \left[ \sum_{t'=t+1}^{t+H} \tilde{c}_{tt'} \tilde{x}_{tt'} \right] \right)$$

$$\begin{split} \tilde{x}_{tt'}^{wd} + \beta \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{gd} &\leq f_{tt'}^{D} \\ \tilde{x}_{tt'}^{gd} + \tilde{x}_{tt'}^{gr} &\leq f_{tt'}^{G} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{rg} &\leq \tilde{R}_{tt'} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{gr} &\leq R^{\max \tilde{x}_{tt'}} \\ \tilde{x}_{tt'}^{wr} + \tilde{x}_{tt'}^{wd} &\leq f_{tt'}^{E} \\ \tilde{x}_{tt'}^{wr} + \tilde{x}_{tt'}^{gr} &\leq \gamma^{ch \arg e} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{rg} &\leq \gamma^{disch \arg e} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{rg} &\leq \gamma^{disch \arg e} \end{split}$$

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Benchmark policy – Deterministic lookahead

$$X_t^{D-LA}(S_t \mid \theta) = \underset{x_t, (\tilde{x}_{tt'}, t'=t+1, \dots, t+H)}{\operatorname{arg min}} \left( C(S_t, x_t) + \left[ \sum_{t'=t+1}^{t+H} \tilde{c}_{tt'} \tilde{x}_{tt'} \right] \right)$$

$$\begin{split} \tilde{x}_{tt'}^{wd} + \beta \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{gd} &\leq f_{tt'}^{D} \\ \tilde{x}_{tt'}^{gd} + \tilde{x}_{tt'}^{gr} &\leq f_{tt'}^{G} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{rg} &\leq \tilde{R}_{tt'} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{gr} &\leq R^{\max \tilde{x}_{tt'}} \\ \tilde{x}_{tt'}^{wr} + \tilde{x}_{tt'}^{wd} &\leq \theta_{t'-t} f_{tt'}^{E} \\ \tilde{x}_{tt'}^{wr} + \tilde{x}_{tt'}^{gr} &\leq \gamma^{ch \arg e} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{rg} &\leq \gamma^{disch \arg e} \end{split}$$

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Benchmark policy – Deterministic lookahead

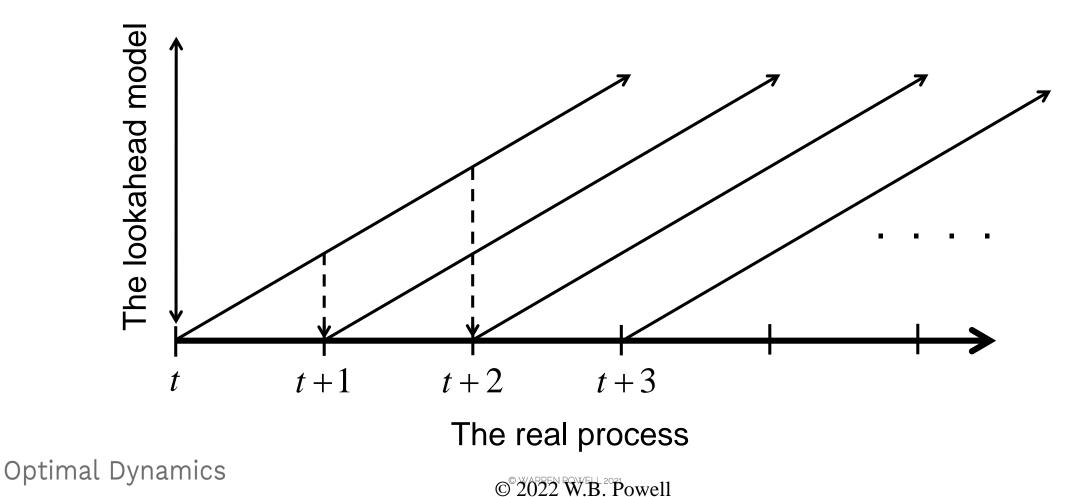
$$\begin{split} X_{t}^{D-LA}(S_{t} \mid \theta) &= \underset{x_{t}, (\tilde{x}_{tt}, t'=t+1, \dots, t+H)}{\operatorname{arg\,min}} \left( C(S_{t}, x_{t}) + \left[ \sum_{t'=t+1}^{t+H} \tilde{c}_{tt}, \tilde{x}_{tt'} \right] \right) \\ \tilde{x}_{tt'}^{wd} + \beta \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{gd} \leq f_{tt'}^{D} \\ \tilde{x}_{tt'}^{gd} + \tilde{x}_{tt'}^{gr} \leq f_{tt'}^{G} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{rg} \leq \tilde{R}_{tt'} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{sgr} \leq R^{\max \tilde{x}_{tt'}} \\ \tilde{x}_{tt'}^{wr} + \tilde{x}_{tt'}^{wd} \leq \theta_{tt-1} f_{tt'}^{E} \\ \tilde{x}_{tt'}^{wr} + \tilde{x}_{tt'}^{sg} \leq \gamma^{\operatorname{charg\,e}} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{rg} \leq \gamma^{\operatorname{charg\,e}} \\ \tilde{x}_{tt'}^{rd} + \tilde{x}_{tt'}^{rg} \leq \gamma^{\operatorname{charg\,e}} \end{split}$$

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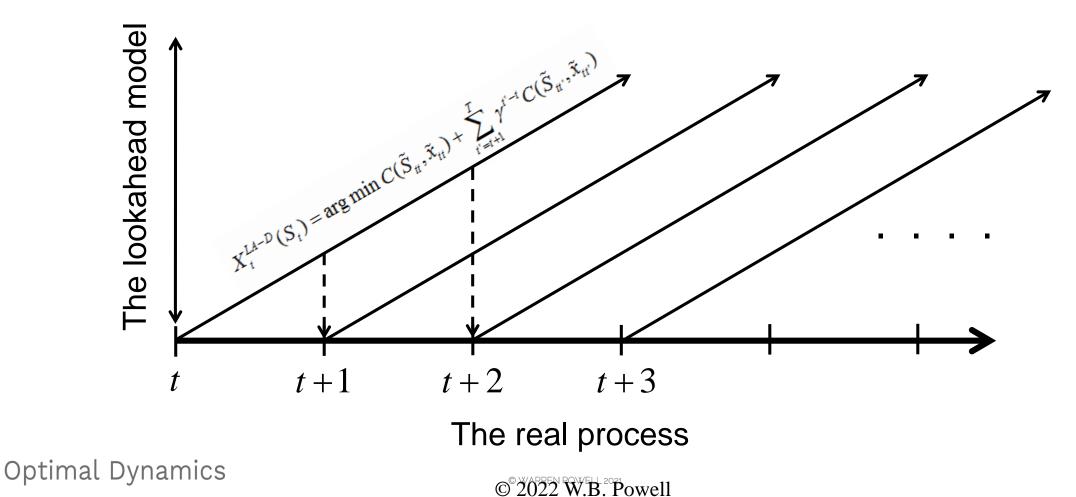


- Lookahead policies peek into the future
  - » Optimize over deterministic lookahead model



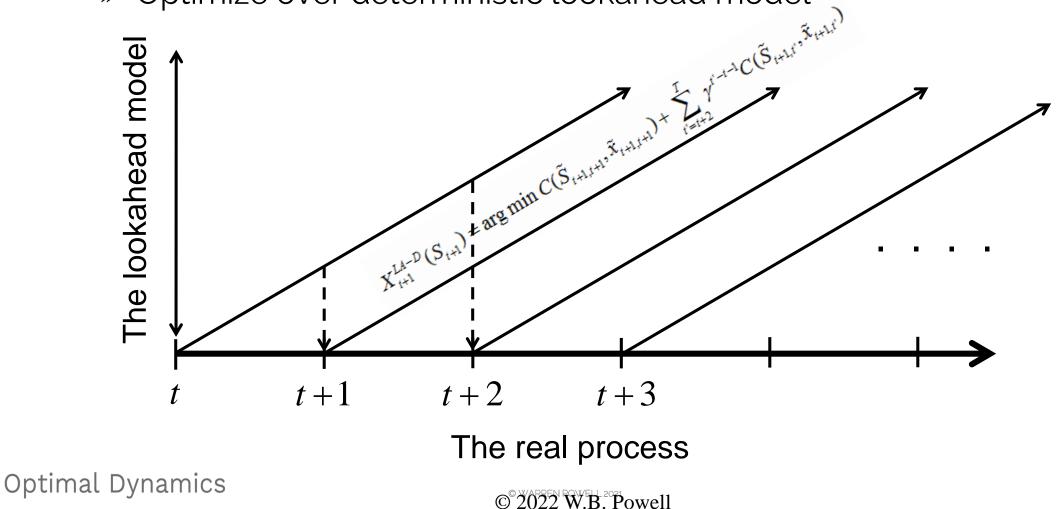


- Lookahead policies peek into the future
  - » Optimize over deterministic lookahead model



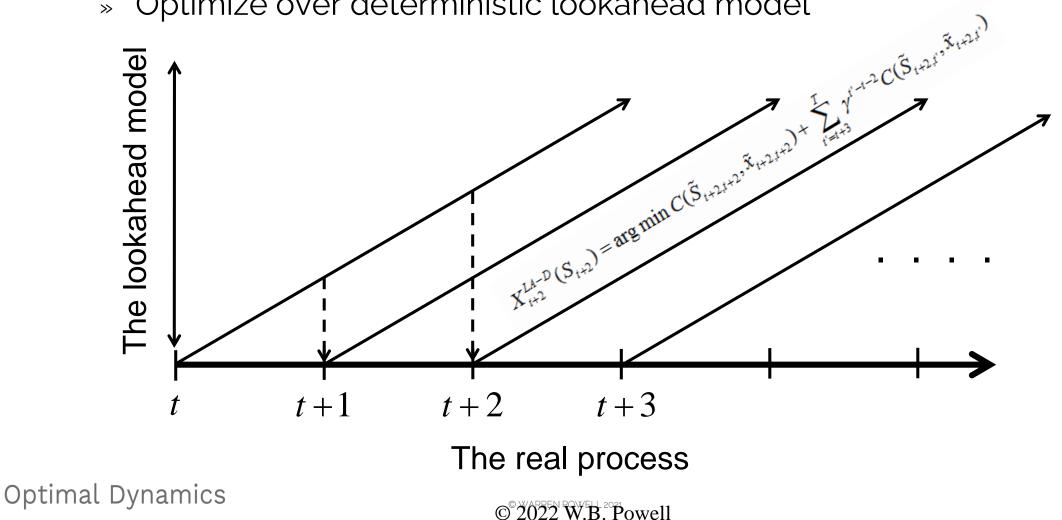


- Lookahead policies peek into the future
  - » Optimize over deterministic lookahead model



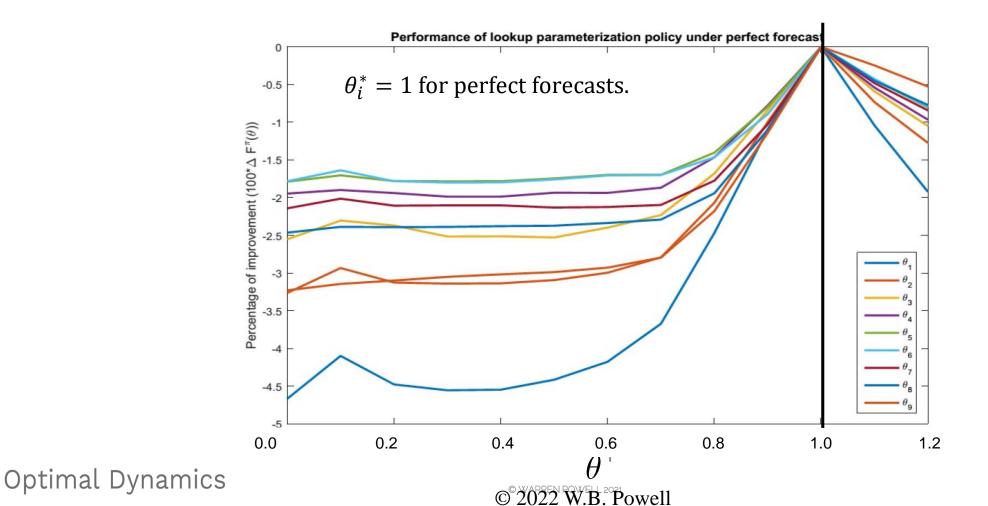


- Lookahead policies peek into the future •
  - » Optimize over deterministic lookahead model



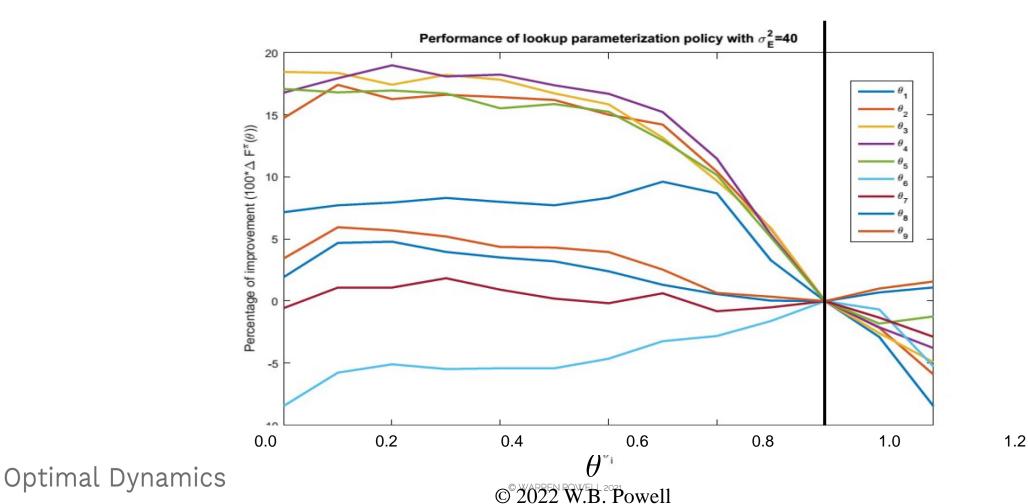


• One-dimensional contour plots – perfect forecast





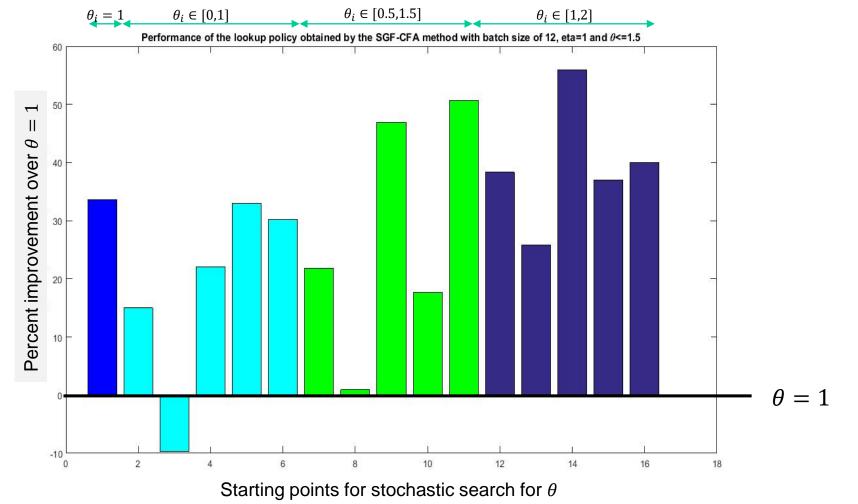
One-dimensional contour plots-uncertain forecast





## **Energy storage optimization**

• Tuning the parameters



() Optimal Dynamics

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## **Cost function approximations**

- Other applications
  - » Airlines optimizing schedules with schedule slack to handle weather uncertainty.
  - » Manufacturers using buffer stocks to hedge against production delays and quality problems.
  - » Grid operators scheduling extra generation capacity in case of outages.
  - » Adding time to a trip planned by Google maps to account for uncertain congestion.

See <u>http://tinyurl.com/cfapolicy</u> for more on parametric cost function approximations.

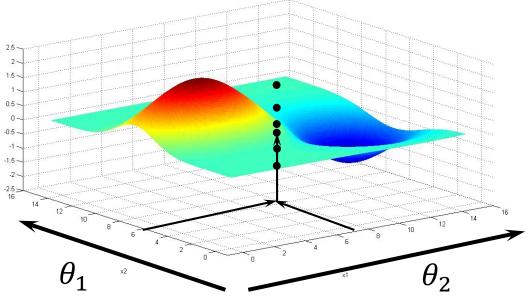


# **Policy search**

- Tuning the policy (PFAs or CFAs):
  - » We need to maximize

$$\max_{\theta} F(\theta) = \mathbb{E} \left\{ \sum_{t=0}^{T} C(S_t, X_t^{\pi}(S_t | \theta)) | S_0 \right\}$$

» We cannot compùte the expectation, so we run simulations:





# **Policy function approximations**

How do we search for the best  $\theta$ ?

» Derivative-based

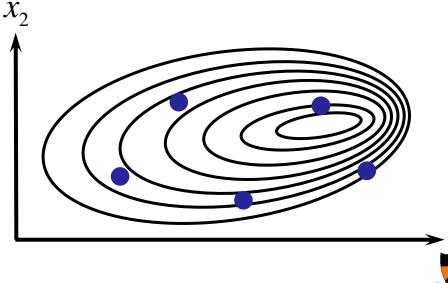
•

Stochastic gradient methods:

 $\theta^{n+1} = \theta^n + \alpha_n \nabla_\theta F(\theta^n, W^{n+1})$ 

 $x_2$ 

- » Derivative-free
  - Build a belief model  $\overline{F}(\theta) \approx \mathbb{E}F(\theta, W)$  that approximates our function.
- » Both of these approaches are sequential decision problems!



Two fundamental strategies for designing policies

**Policy search** – Search over a class of methods for making decisions to optimize some metric over time.

- » Finding the best class of policy.
- » Finding the best policy within the class.

**Lookahead approximations** – Approximate the impact of a decision now on the future.

- » The contribution from the first period, plus
- » An approximation of the sum of contributions in future time periods resulting from the first decision.



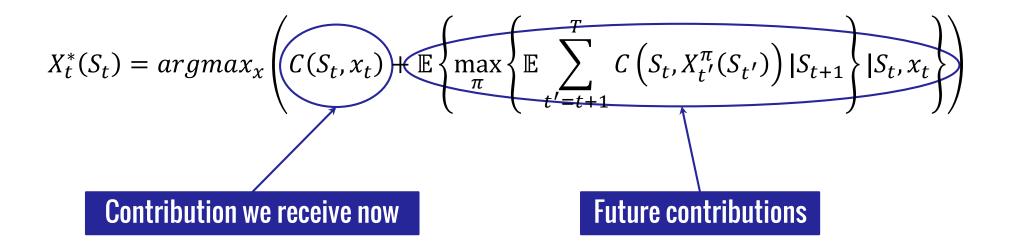
### Lookahead approximations

- Lookahead approximations combine:
  - » The immediate contribution (or cost) of a decision made now...
  - » ... and an approximation of future contributions (or costs)



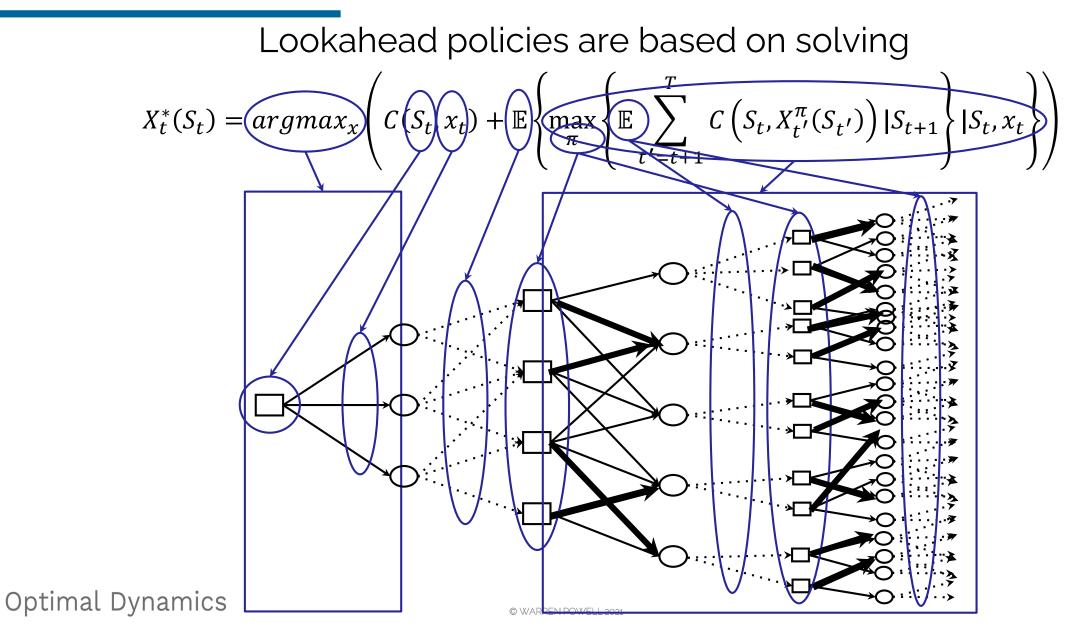


Lookahead policies are based on solving



- » This looks like scary mathematics, but it is what all of us are doing when we make decisions now that consider what might happen in the future.
- » The challenge is ... *how to compute it!!!*







#### Lookahead approximations

Approximate the impact of a decision now on the future

$$X_{t}^{*}(S_{t}) = \arg\max_{x} \left( C(S_{t}, x_{t}) + \mathbb{E} \left\{ \max_{\pi} \left\{ \mathbb{E} \sum_{t'=t+1}^{T} C\left(S_{t}, X_{t'}^{\pi}(S_{t'})\right) | S_{t+1} \right\} | S_{t}, x_{t} \right\} \right)$$

$$3) Value function approximations (VFAs)$$

$$X_{t}^{*}(S_{t}) = \arg\max_{x_{t}} \left( C(S_{t}, x_{t}) + \mathbb{E} \left\{ V_{t+1}(S_{t+1}) | S_{t}, x_{t} \right\} \right)$$

$$X_{t}^{VFA}(S_{t}) = \arg\max_{x_{t}} \left( C(S_{t}, x_{t}) + \mathbb{E} \left\{ \overline{V}_{t+1}(S_{t+1}) | S_{t}, x_{t} \right\} \right)$$

$$= \arg\max_{x_{t}} \left( C(S_{t}, x_{t}) + \overline{V}_{t}^{x}(S_{t}^{x}) \right)$$

$$= \arg\max_{x_{t}} \left( \overline{Q}_{t}(S_{t}, x_{t}) + \overline{V}_{t}^{x}(S_{t}^{x}) \right)$$



Processes

ming

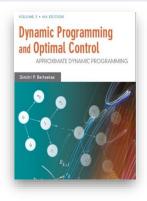
#### Lookahead approximations

Approximate the impact of a decision now on the future

$$X_t^*(S_t) = \operatorname{argmax}_x \left( C(S_t, x_t) + \mathbb{E} \left\{ \max_{\pi} \left\{ \mathbb{E} \sum_{t'=t+1}^T C\left(S_t, X_{t'}^{\pi}(S_{t'})\right) | S_{t+1} \right\} | S_t, x_t \right\} \right)$$

3) Value function approximations (VFAs)

$$\begin{aligned} X_t^*(S_t) &= \arg\max_{x_t} \left( C(S_t, x_t) + \mathsf{E}\left\{ V_{t+1}(S_{t+1}) \mid S_t, x_t \right\} \right) \\ X_t^{VFA}(S_t) &= \arg\max_{x_t} \left( C(S_t, x_t) + \mathsf{E}\left( \overline{V_{t+1}}(S_{t+1}) \mid S_t, x_t \right\} \right) \\ &= \arg\max_{x_t} \left( C(S_t, x_t) + \overline{V_t}^x(S_t^x) \right) \\ &= \arg\max_{x_t} \left( \overline{Q_t}(S_t, x_t) - ("Q-\text{learning"}) \right) \end{aligned}$$





#### Lookahead approximations

Approximate the impact of a decision now on the future

$$X_{t}^{*}(S_{t}) = \arg\max_{x} \left( C(S_{t}, x_{t}) + \mathbb{E} \left\{ \max_{\pi} \left\{ \mathbb{E} \sum_{t'=t+1}^{T} C\left(S_{t}, X_{t'}^{\pi}(S_{t'})\right) | S_{t+1} \right\} | S_{t}, x_{t} \right\} \right)$$
  
**3) Value function approximations (VFAs)**  

$$X_{t}^{*}(S_{t}) = \arg\max_{x_{t}} \left( C(S_{t}, x_{t}) + \mathbb{E} \left\{ V_{t+1}(S_{t+1}) | S_{t}, x_{t} \right\} \right)$$
  

$$X_{t}^{VFA}(S_{t}) = \arg\max_{x_{t}} \left( C(S_{t}, x_{t}) + \mathbb{E} \left\{ \overline{V}_{t+1}(S_{t+1}) | S_{t}, x_{t} \right\} \right)$$
  

$$= \arg\max_{x_{t}} \left( C(S_{t}, x_{t}) + \overline{V}_{t}^{x}(S_{t}^{x}) \right)$$
  

$$= \arg\max_{x_{t}} \overline{Q}_{t}(S_{t}, x_{t}) \quad ("Q-learning")$$

pproximate Dynamic Programming Solving the Carses of Dimensionality

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#### Lookahead approximations

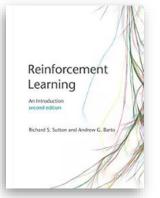
Approximate the impact of a decision now on the future

$$X_t^*(S_t) = \arg\max_x \left( C(S_t, x_t) + \mathbb{E} \left\{ \max_{\pi} \left\{ \mathbb{E} \sum_{t'=t+1}^T C\left(S_t, X_{t'}^{\pi}(S_{t'})\right) | S_{t+1} \right\} | S_t, x_t \right\} \right)$$

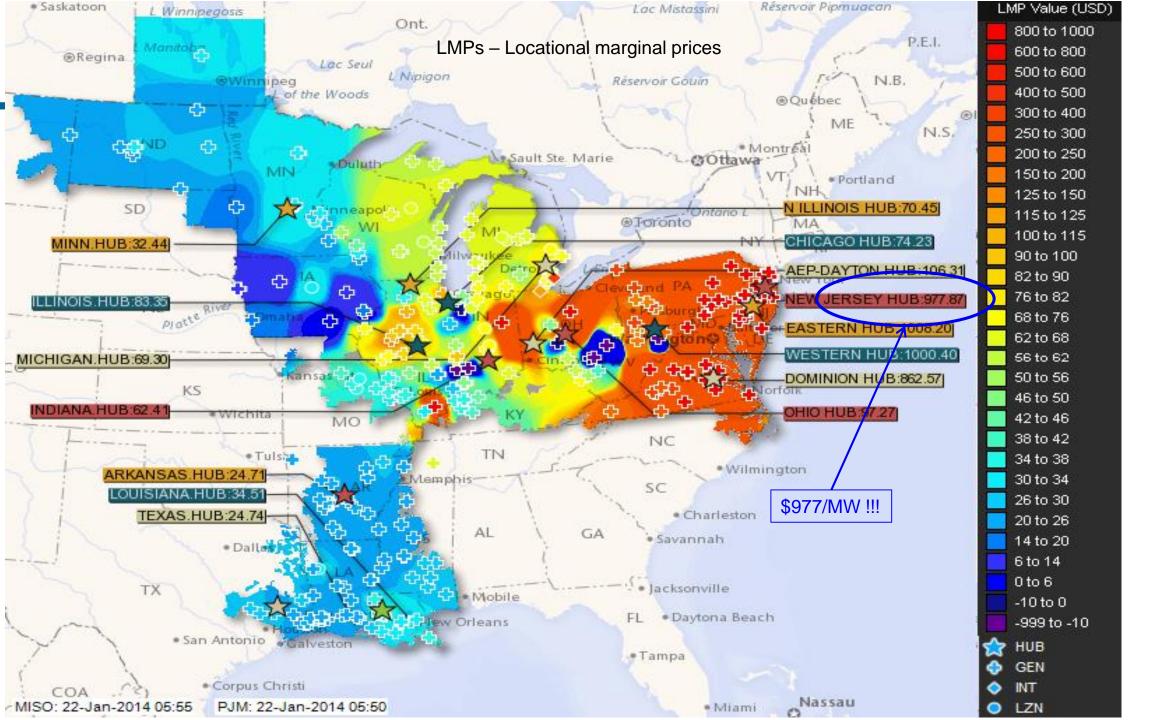
3) Value function approximations (VFAs)

$$\begin{aligned} X_t^*(S_t) &= \arg\max_{x_t} \left( C(S_t, x_t) + \mathsf{E}\left\{ V_{t+1}(S_{t+1}) \mid S_t, x_t \right\} \right) \\ X_t^{VFA}(S_t) &= \arg\max_{x_t} \left( C(S_t, x_t) + \mathsf{E}\left\{ \overline{V}_{t+1}(S_{t+1}) \mid S_t, x_t \right\} \right) \\ &= \arg\max_{x_t} \left( C(S_t, x_t) + \overline{V}_t^x(S_t^x) \right) \\ &= \arg\max_{x_t} \left( \overline{Q}_t(S_t, x_t) \right) \text{ ("Q-learning")} \end{aligned}$$

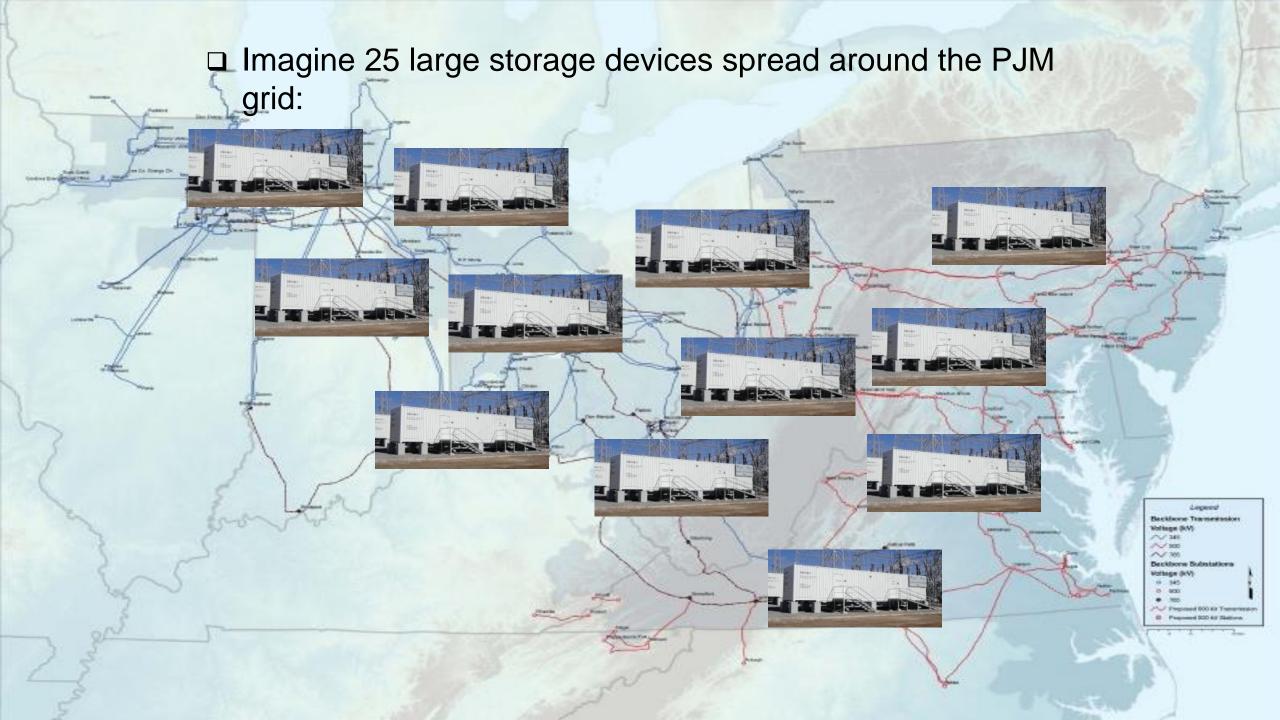
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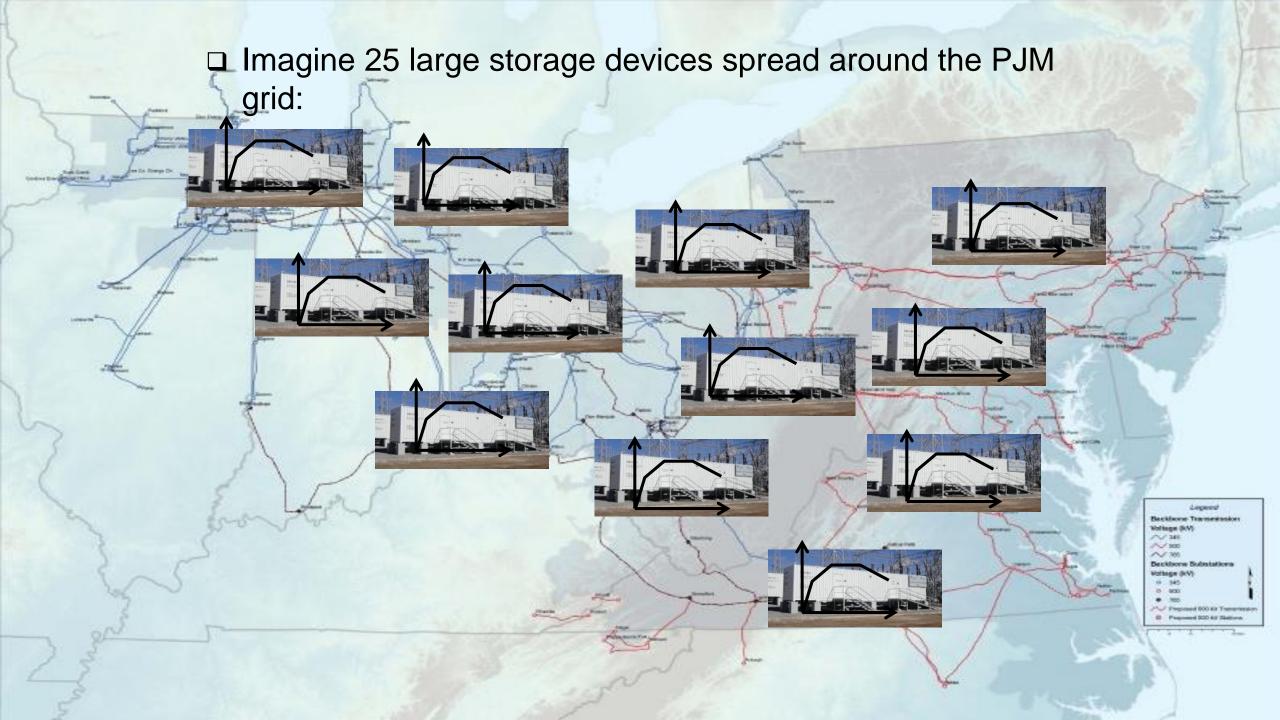


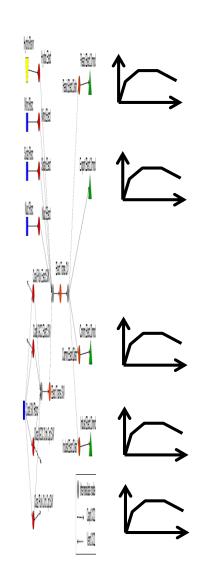
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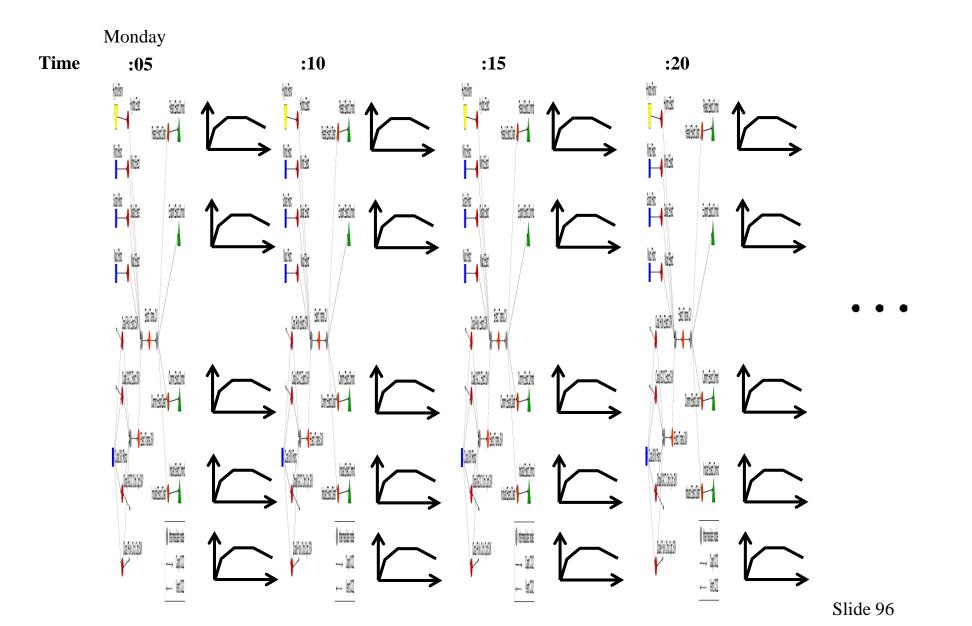


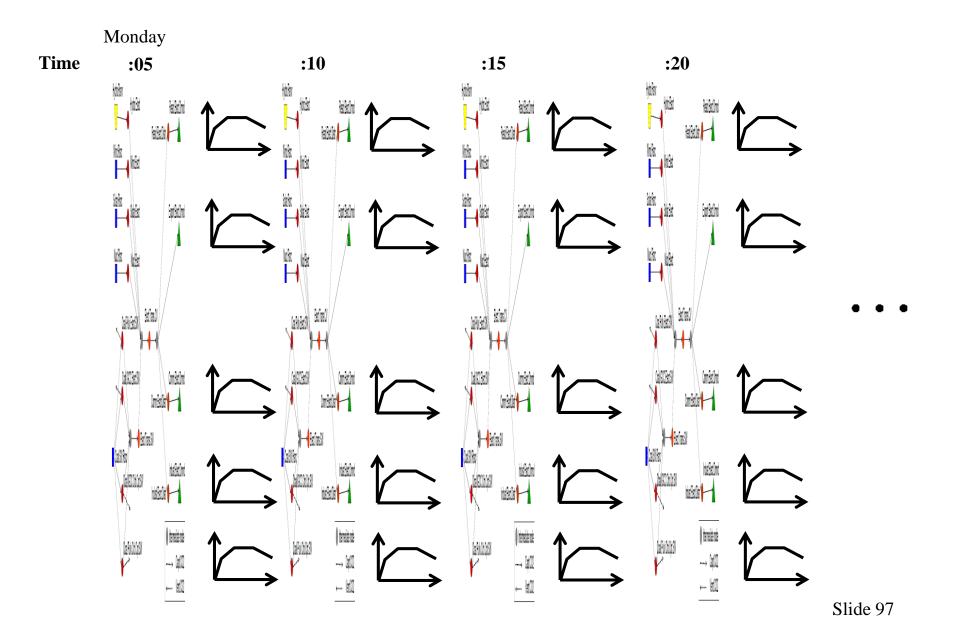


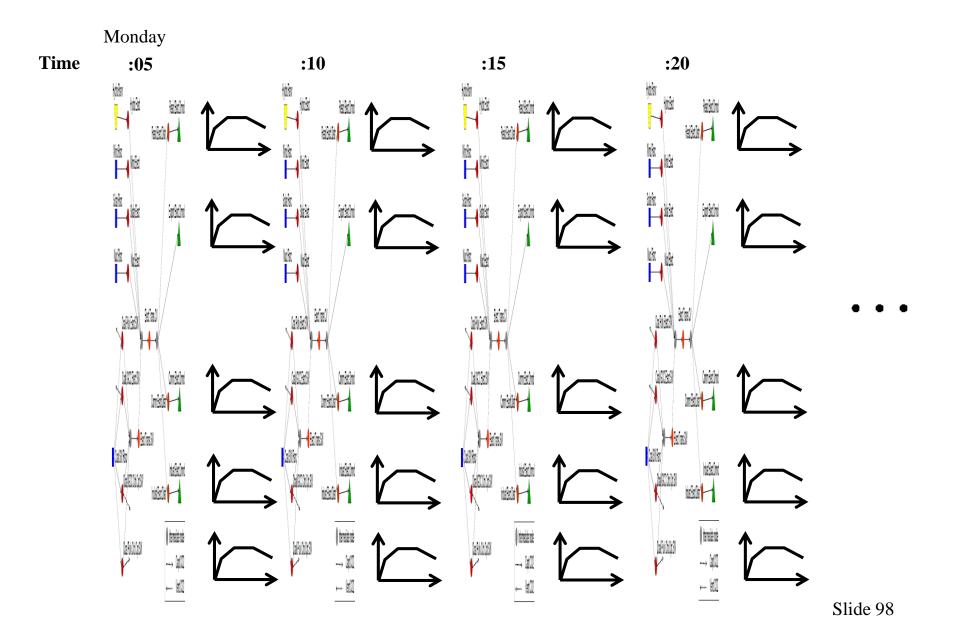




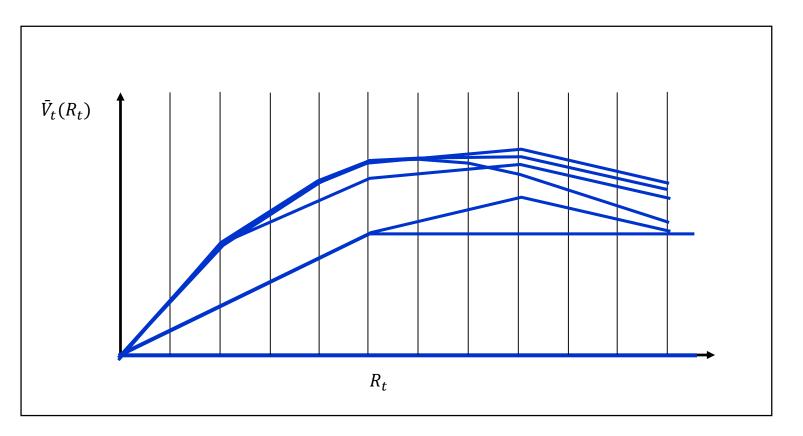






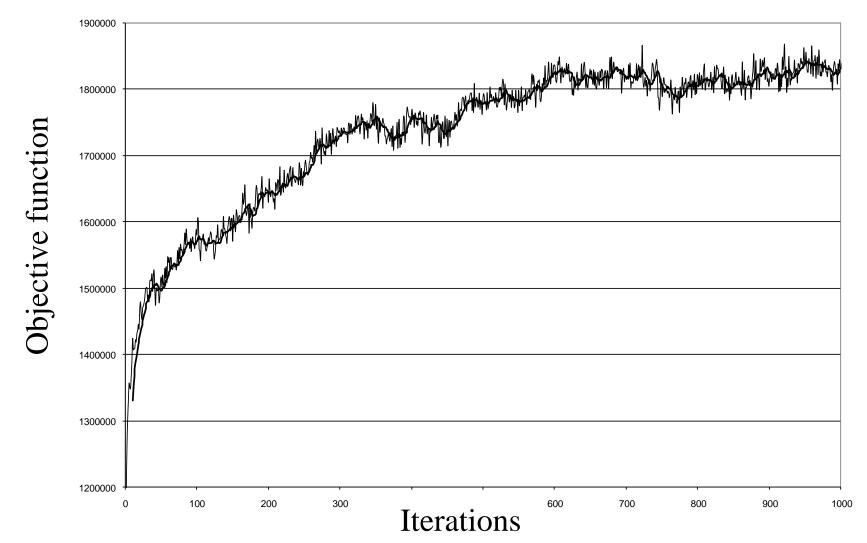


 Derivatives are used to estimate a piecewise linear approximation

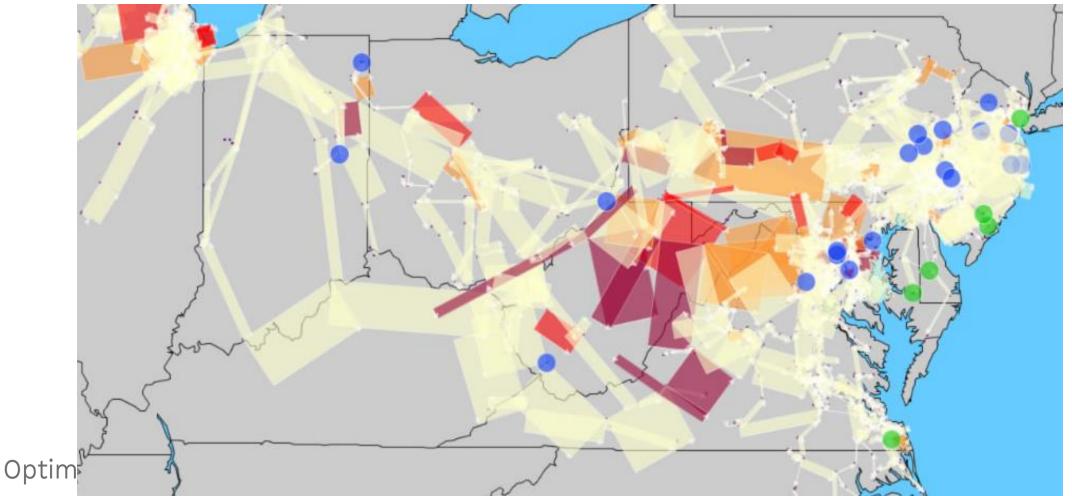




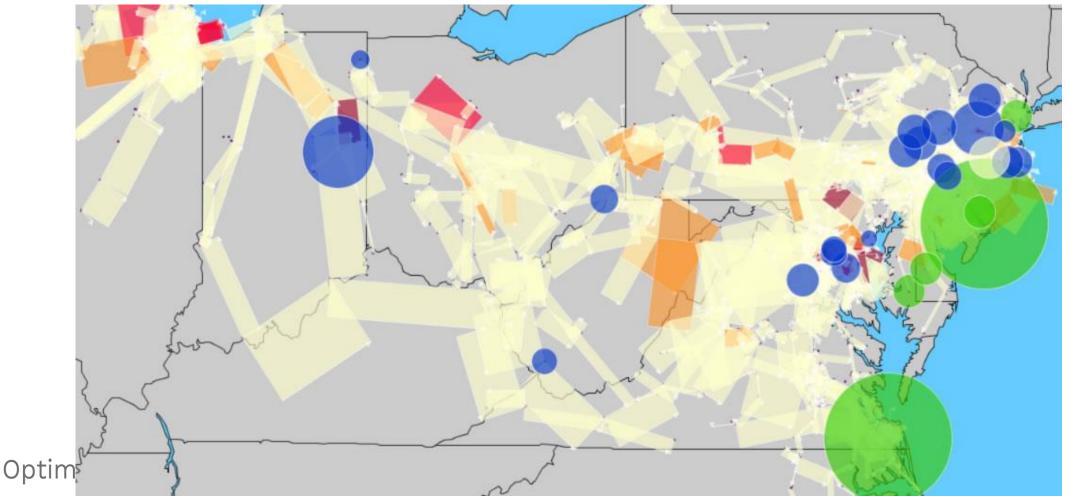
• With luck, your objective function improves



- Congested grid:
  - » Green and blue circles indicate energy storage

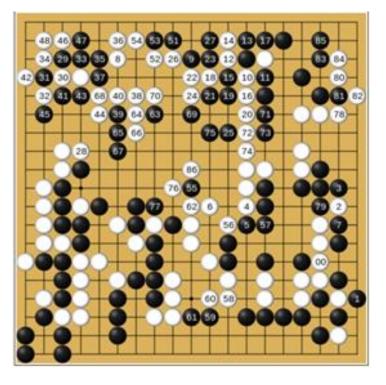


- Congested grid:
  - » Green and blue circles indicate energy storage



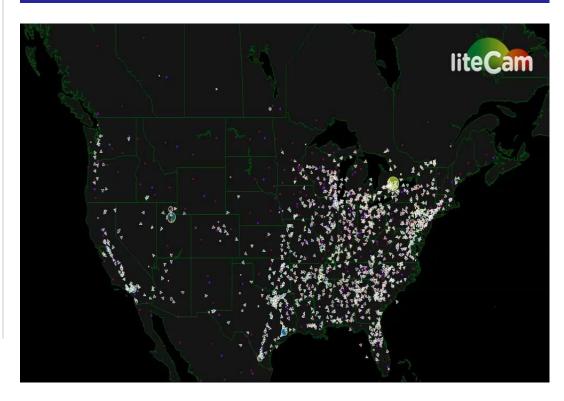
### **NOTABLE APPLICATIONS**

#### **Reinforcement Learning**



» Major achievement — Playing Go

#### **High-Dimensional ADP**



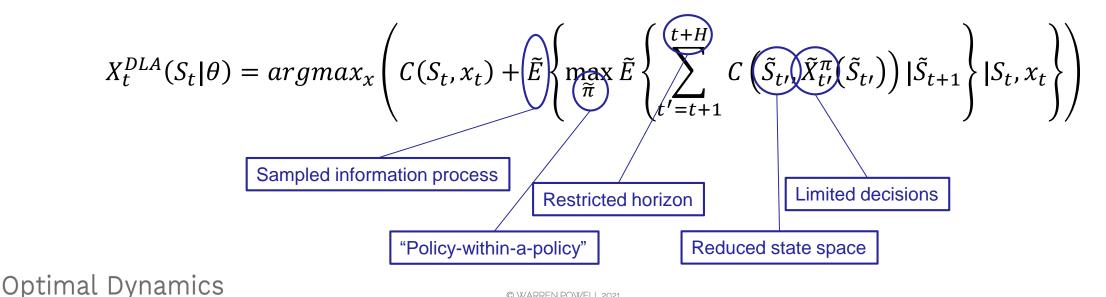
» Major achievement — Optimizing a 5000 truck fleet



4) Direct lookahead policies (DLAs) – Here we create an approximation called the *approximate lookahead model*:

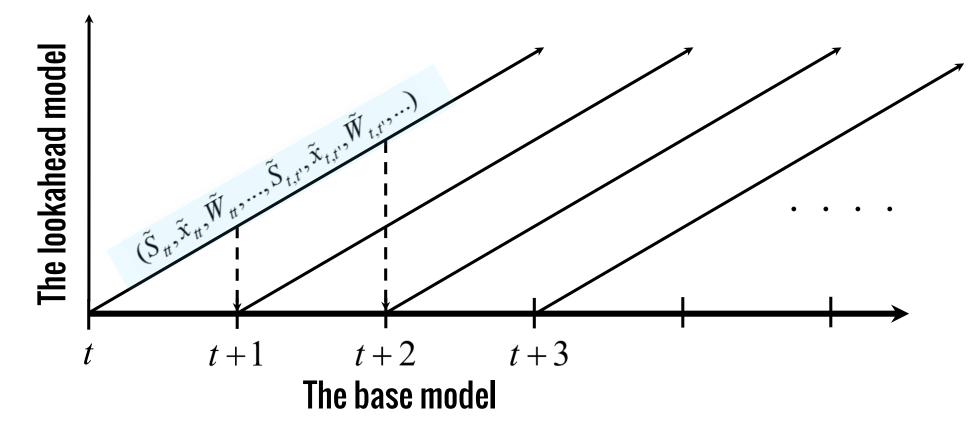
$$(\tilde{S}_{tt}, \tilde{x}_{tt}, \tilde{W}_{t,t+1}, \tilde{S}_{t,t+1}, \tilde{x}_{t,t+1}, \tilde{W}_{t,t+2}, \dots, \tilde{S}_{tt'}, \tilde{x}_{tt'}, \tilde{W}_{t,t'+1}, \dots)$$

There are six classes of approximations we can introduce. Our direct lookahead policy now requires solving:



Direct Lookahead Policies (DLAs)

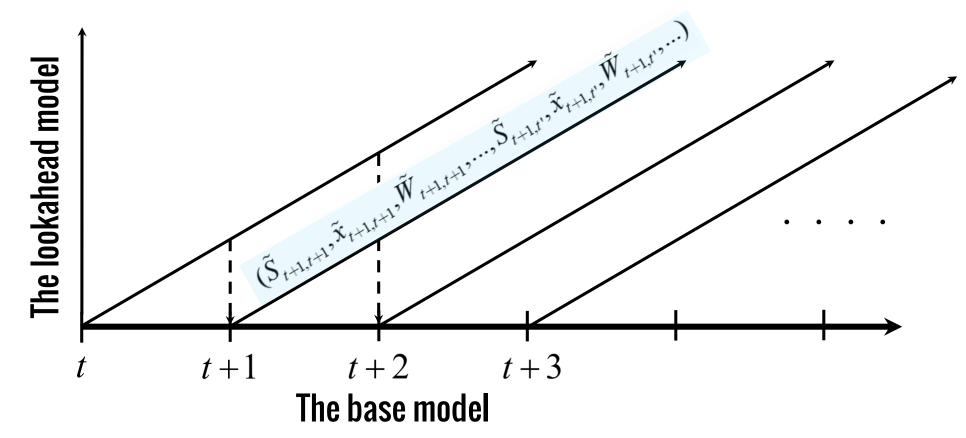
» Tilde variables are used to model approximate lookahead





Direct Lookahead Policies (DLAs)

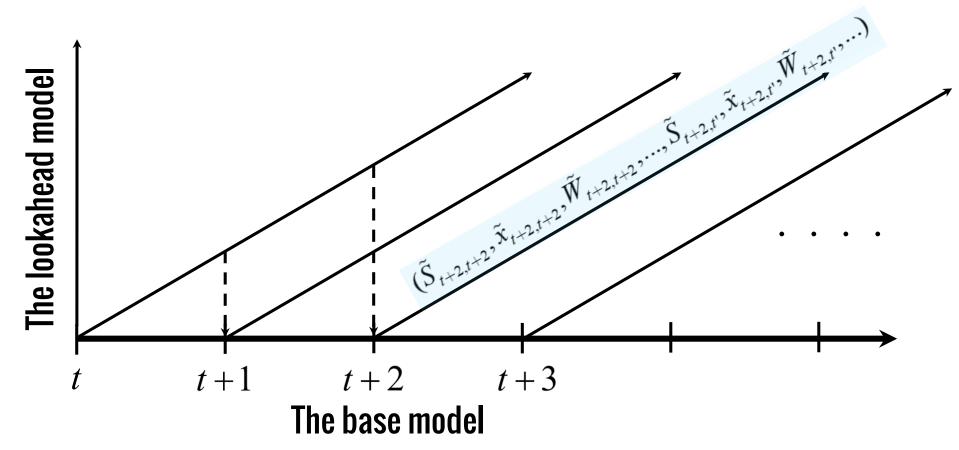
» Tilde variables are used to model approximate lookahead





Direct Lookahead Policies (DLAs)

» Tilde variables are used to model approximate lookahead

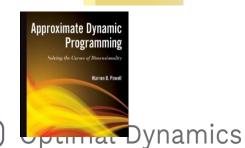






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Laurent El Ghaoui	
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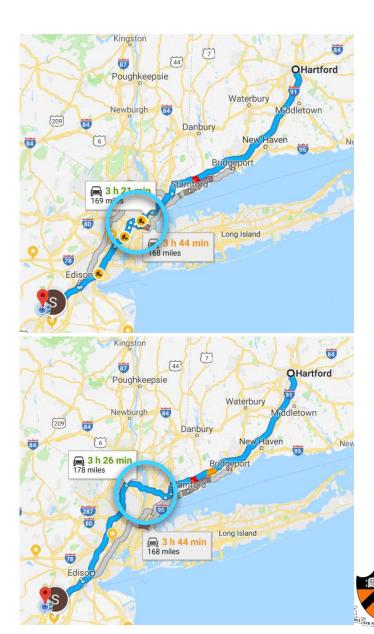


### Examples of Lookahead Models

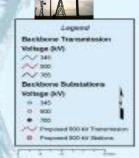
#### » The deterministic lookahead model

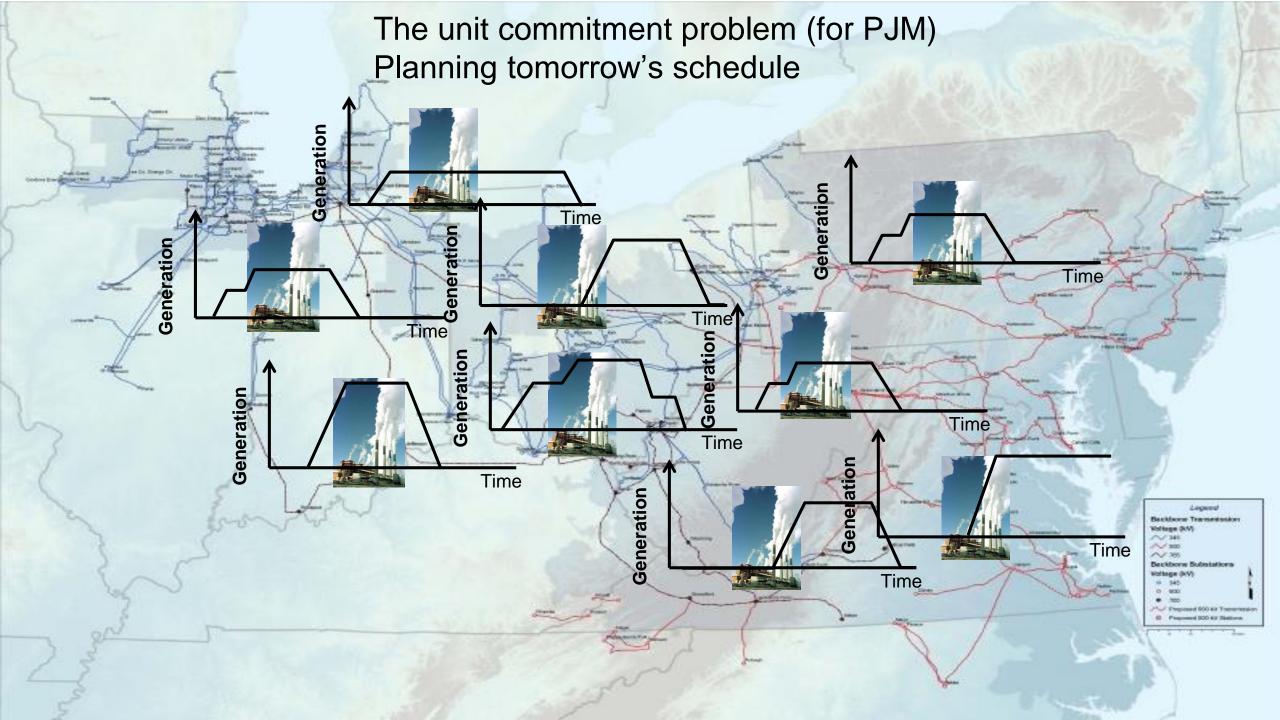
- This is what is most widely used in practice.
- Standard approach is to use a "best estimate" (which means deterministic) of travel times in the future.
- This is often referred to as "model predictive control"
- » Robust optimization We could use the 90<sup>th</sup> percentile of travel times.
- Stochastic programming We represent the future using, say, 20 samples.
- Approximate dynamic programming applied to approximate lookahead model
- Chance constrained programming Impose constraint on the probability of being late.

See Chapter 19 at http://tinyurl.com/RLandSO © WARREN POWELL 2021



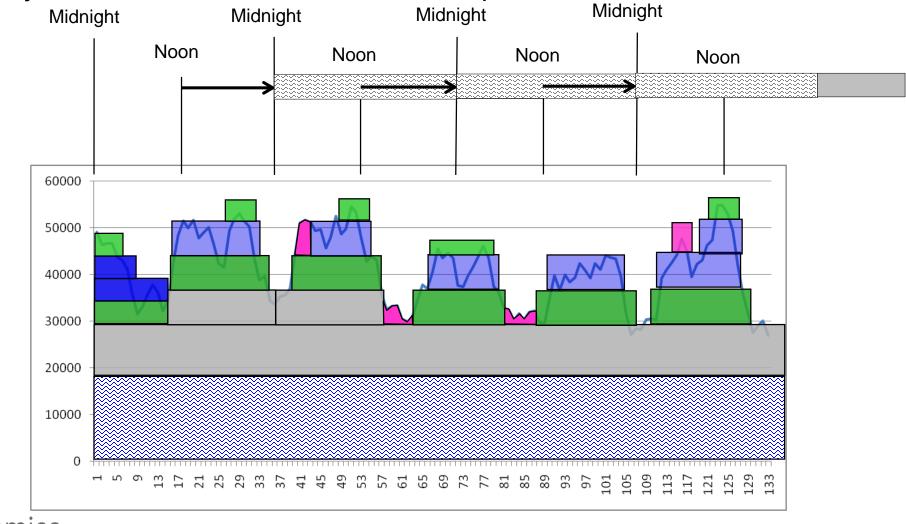
The unit commitment problem (for PJM) Planning tomorrow's schedule





## The timing of decisions

• The day-ahead unit commitment problem

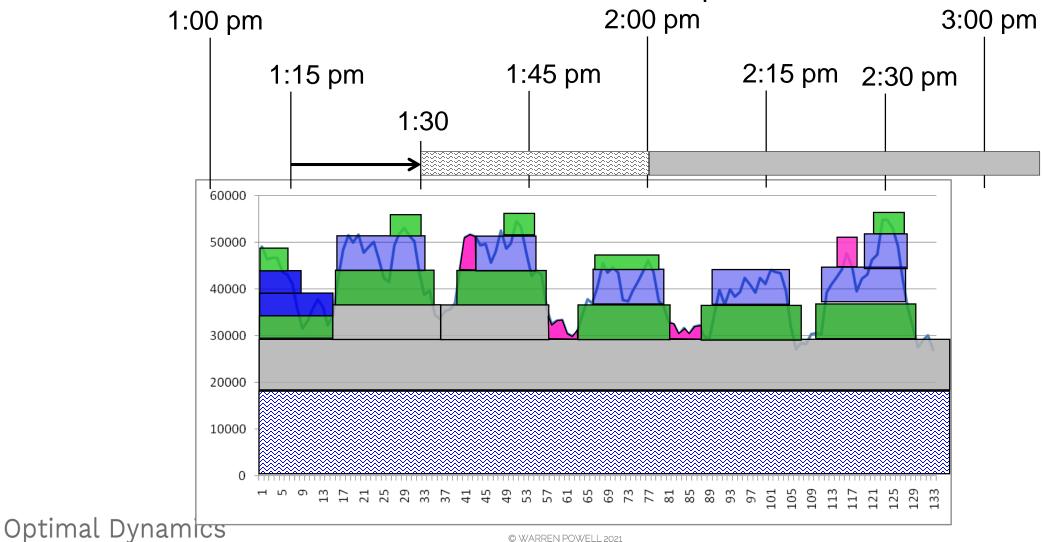




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## The timing of decisions

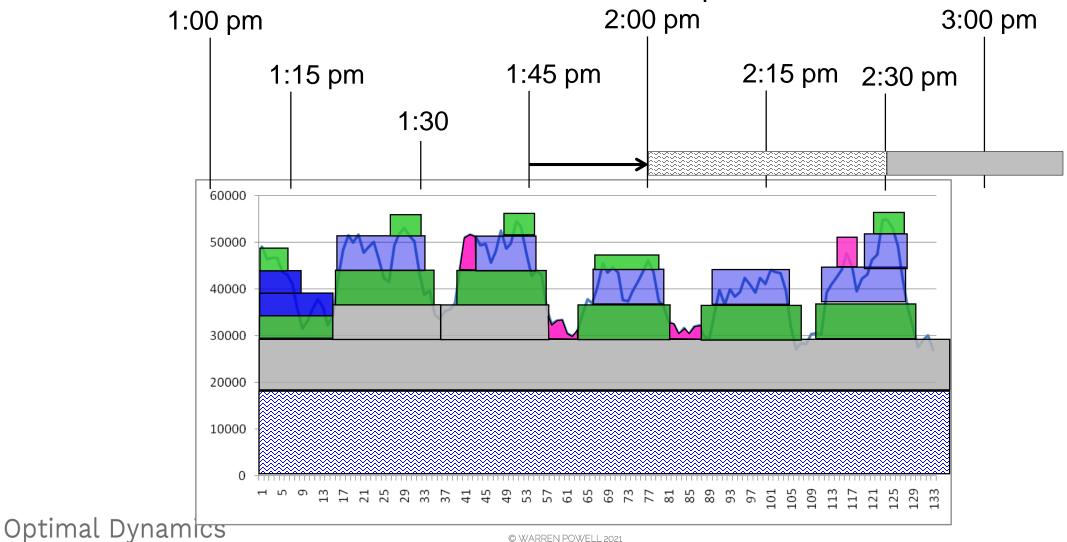
Intermediate-term unit commitment problem





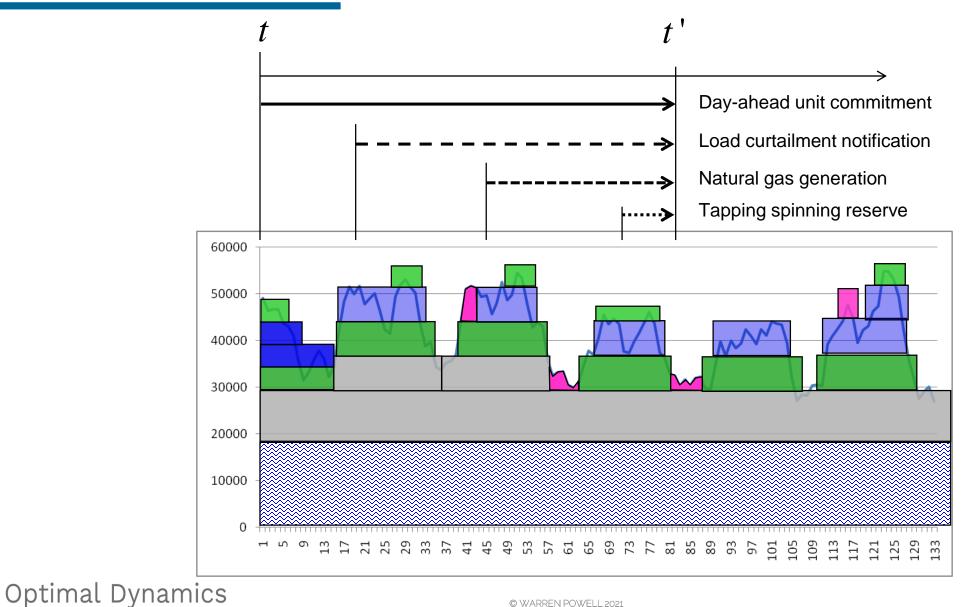
## The timing of decisions

Intermediate-term unit commitment problem



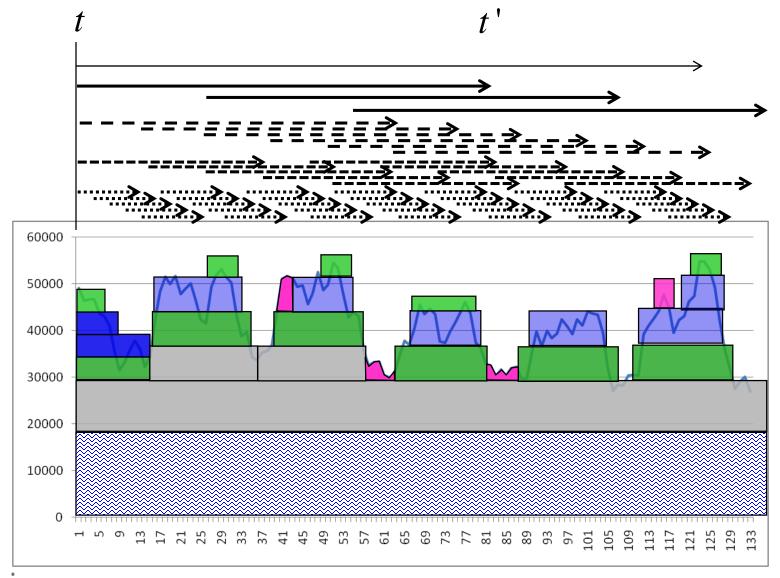


## The unit commitment problem





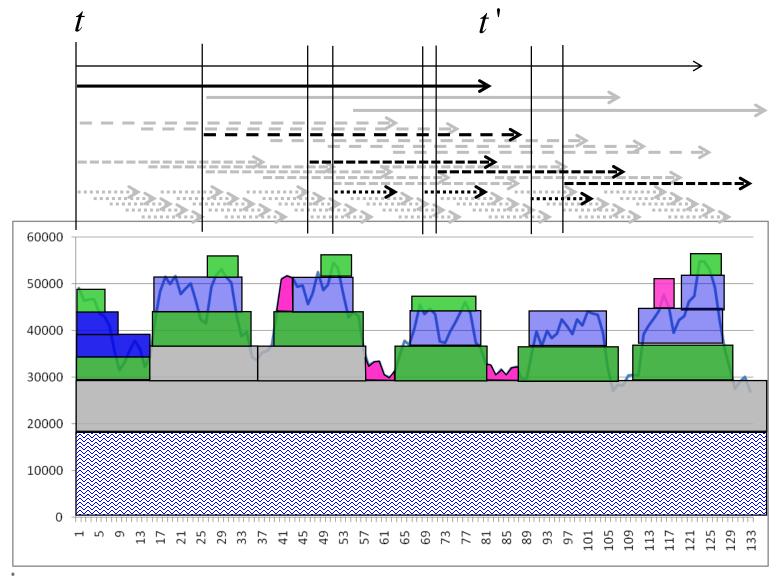
## The unit commitment problem





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## The unit commitment problem





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## **DESIGNING POLICIES**

#### Policy search policies

#### Policy function approximations (PFAs)

- » Simple rules, functions
- » Examples:
  - Order up to
  - Buy low, sell high

#### Lookahead policies

#### Value function approximations (VFAs)

- » Making a decision now using the value of being in a future state
- » Examples:
  - The value of a truck driver
  - The value of holding an asset

#### **Cost function approximations (CFAs)**

- » Parameterized cost models
- » Examples
  - Schedule slack for trips
  - Buffer stocks for inventory

#### **Direct lookaheads (DLAs)**

- Models that optimize over a planning horizon (deterministically/stochastically)
- » Examples:
  - Google maps
  - Energy planning models



The four classes of policies are *universal* – they cover every method for making decisions described in the research literature or used in practice.

This means you are already using one of the four classes of policies (or a hybrid) in your own decisions.But are you doing the best you can?

#### THE UNIVERSAL FRAMEWORK FOR SEQUENTIAL **DECISIONS** Warren B. Powell, Princeton University http://tinyurl.com/powelljungle **Policy Function Approximation(PFA)** $\max_{\pi} \mathbb{E} \left\{ \sum_{t=0}^{l} C(S_t, X^{\pi}(S_t)) | S_0 \right\}$ If this then do $X^{PFA}(S_t|\theta) = \begin{cases} \sum_{f \in F} \theta_f \phi_f(S_t) \end{cases}$ where $S_{t+1} = S^{M}(S_{t}, X^{\pi}(S_{t}), W_{t+1})$ eural network Policy Search and given $(S_0, W_1, W_2, \dots, W_t, \dots)$ **Cost Function Approximation (CFA)** $X^{CFA}(S_t|\theta) = \begin{cases} argmax_x c_t x_t + \sum_f \theta_f \phi_f(S_t) \\ argmax_x (\mu_{tx} + \theta^{IE} \bar{\sigma}_{tx}) \end{cases}$ Policy Lookahead Approximations Value Function Approximation (VFA) The four classes of policies (PFAs, CFAs, VFAs and DLAs) are universal. $X^{VFA}(S_t|\theta) = argmax_x(C(S_t, x) + \mathbb{E}\{V_{t+1}(S_{t+1})|S_t, x_t\})$ $= argmax_{x}(C(S_{t}, x) + \overline{V}_{t}^{x}(S_{t}^{x}))$ Any sequential decision problem will use $= argmax Q(S_t, x)$ one of these four classes (or a hybrid), including whatever you might be doing now. **Direct Lookahead (DLA)** The optimal policy (if we could solve it) is given by $X^{DLA}(S_t|\theta) = argmax_x \left( c_t x_t + \sum_{t'=t+1}^{t+H} \tilde{c}_{tt'} \tilde{x}_{tt'} \right)$ $X^*(S_t) = argmax_x \left( C(S_t, x) + \mathbb{E} \left\{ \max_{\pi} \mathbb{E} \left\{ \sum_{t'=t+1}^{t+H} C(S_{t'}, X^{\pi}(S_{t'})) | S_{t+1} \right\} | S_t, x_t \right\} \right)$

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## **DESIGNING POLICIES**

> Policies in the "policy search" class are simplest

 As a result, this is what you are most likely going to see used in practice.

... but ...

• "The price of simplicity is tunable parameters"

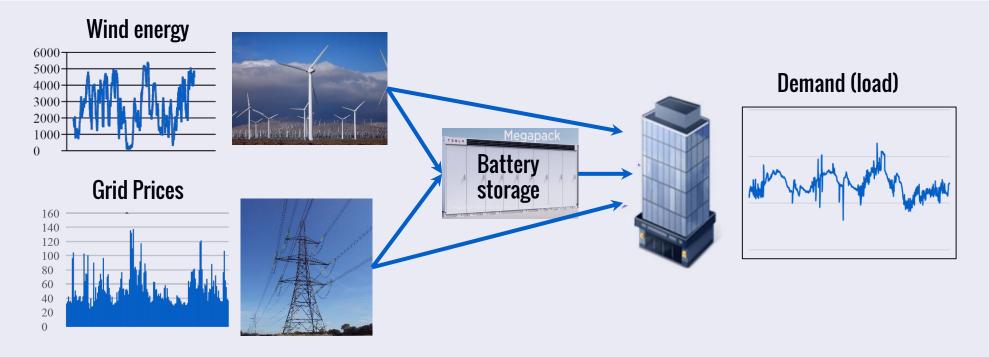
... and ...

• "Tuning is hard"!



## **AN ENERGY STORAGE PROBLEM**

#### Consider a basic energy storage problem



We are going to show that with minor variations in the characteristics of this problem, we can make *each* class of policy work best.



## **AN ENERGY STORAGE PROBLEM**

#### Each policy is best on certain problems

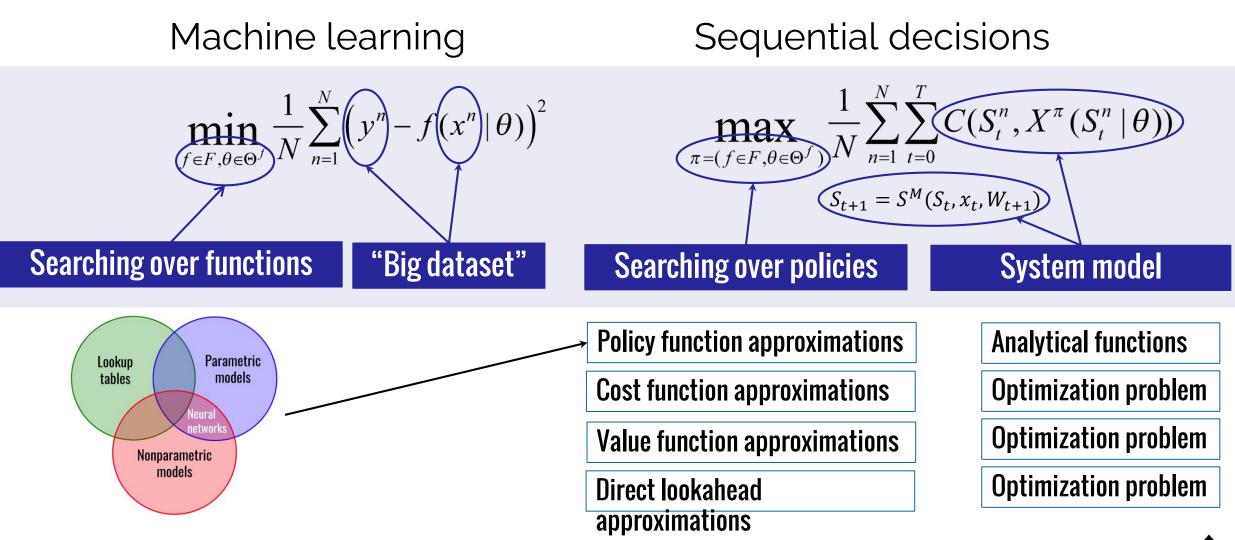
Problem:	Problem description	PFA	CFA Error correction	VFA	Determ. Lookahead	CFA Lookahead
А	A stationary problem with heavy-tailed prices, relatively low noise, moderately accurate forecasts.	0.959	0.839	0.936	0.887	0.887
В	A time-dependent problem with daily load patterns, no seasonalities in energy and price, relatively low noise, less accurate forecasts.	0.714	0.752	0.712	0.746	0.746
с	A time-dependent problem with daily load, energy and price patterns, relatively high noise, forecast errors increase over horizon.	0.865	0.590	0.914	0.886	0.886
D	A time-dependent problem, relatively low noise, very accurate forecasts.	0.962	0.749	0.971	0.997	0.997
E	Same as (C), but the forecast errors are stationary over the planning horizon.	0.865	0.590	0.914	0.922	0.934

Joint research with Prof. Stephan Meisel, University of Muenster, Germany.

» ... any policy might be best depending on the data.



### **BRIDGING MACHINE LEARNING & SEQUENTIAL DECISIONS**



() Optimal Dynamics

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## Choosing a policy class

#### A new book:

- » First book to introduce a universal modeling framework, covering all four classes of policies.
- Describes the tools for modeling and solving any sequential decision problem, from simple learning problems to truckload fleets to complex supply chains.
- Aimed at a technical audience interested in writing software to develop models such as those described in this presentation.
- Provides the foundation for a new field we are calling sequential decision analytics.
- » To appear May, 2022.

#### http://tinyurl.com/RLandSO/

() Optimal Dynamics

#### REINFORCEMENT LEARNING AND STOCHASTIC OPTIMIZATION

A UNIFIED FRAMEWORK FOR SEQUENTIAL DECISIONS

WARREN B. POWELL

## Choosing a policy class

#### An introductory book:

- » Uses a teach-by-example style
- Illustrates how to model sequential decision problems using a rich set of examples
- » Illustrates all four classes of policies
- » Highlights uncertainty modeling

<u>http://</u>tinyurl.com/sdaexamplesprint

() Optimal Dynamics

Foundations and Trends<sup>®</sup> in Technology, Information and Operations Management Sequential Decision Analytics and Modeling:

Modeling with Python

Suggested Citation: Warren Powell (2022), "Sequential Decision Analytics and Modeling:", Foundations and Trends<sup>®</sup> in Technology, Information and Operations Management: Vol. xx, No. xx, pp 1–. DOI: /XXXXXXXX.

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### OUTLINE

→ The five layers of intelligence

- → Modeling sequential decision problems
- → Designing policies
  - → Policy function approximations
  - $\rightarrow$  Cost function approximations
  - $\rightarrow$  Value function approximations
  - → Direct lookahead approximations

→ A new educational field: sequential decision analytics

### The core disciplines of decision analytics

Optimization

Each of these fields have well-defined communities, using common notation and established tools.

There are widely used textbooks that cover common material, with standard notational frameworks..

Programming and Economic

OPTIMIZATION

Linear Programmi and Network Flov

Analysis

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linear

LINEAR

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Simulation



The concepts are taught in hundreds of academic programs, producing thousands of graduates each year which can be hired by industry.



## **OPTIMIZATION UNDER UNCERTAINTY**

# The fields that deal with decisions and uncertainty are completely fragmented.

- Sequential decision analytics is not a recognized field.
- There are 15 distinct communities that deal with decisions under uncertainty
- Each community offers tools that work only for narrowly defined problem classes.
- Real applications require skills that span a wide range of problem settings.





## **OPTIMIZATION UNDER UNCERTAINTY**

# The fields that deal with decisions and uncertainty are completely fragmented.

- Sequential decision analytics is not a recognized field.
- There are 15 distinct communities that deal with decisions under uncertainty
- Each community offers tools that work only for specific problems
- Real applications require skills that span a wide range of problem settings.



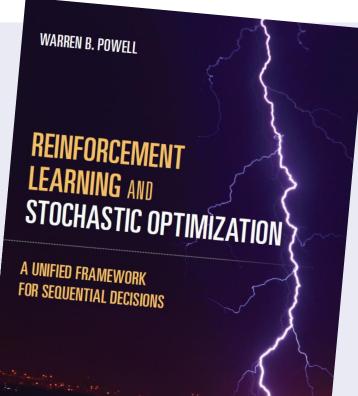
These will be the first books to present sequential decision problems and solution methods in a unified way.



## **SEQUENTIAL DECISION ANALYTICS**

#### An academic proposal

- » We need to establish academic programs in engineering focusing broadly on sequential decision analytics, comparable to existing programs in machine learning.
- » As with machine learning, this program could be centered on methodology, or different problem domains that span engineering (all fields), the sciences (all fields), business, finance, logistics, energy and health.



WILEY

FOR MORE INFORMATION, VISIT http://tinyurl.com/sdafield

**Optimal Dynamics** 



## Thank you!

See

http://tinyurl.com/sdafield

for an introduction to a field I am calling

"Sequential Decision Analytics"

My new book is available at

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